Numerical and Experimental Investigation of Impulse-Radiating Antennas for Use in Sensing Applications

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Kangwook Kim

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Approved by:

Waymond R. Scott, Jr., Chairman

Glenn S. Smith

Andrew F. Peterson

W. Marshall Leach

J. Carlos Santamarina

Date Approved __________________
To my parents, Cheonsoon Kim and Kwangshim Choi

to my wife, Kyongok Hong

and to my son, Matthew Kangwook Kim.
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SUMMARY

The objective of this dissertation was to investigate two types of pulse-radiating antennas for use in remote sensing applications. The antennas investigated in this dissertation were the impulse-radiating antenna (IRA) and the resistive vee dipole (RVD). The characteristics of these antennas were investigated using numerical, experimental, and analytical models.

First, the IRAs were categorized to lens-type and reflector-type, and the analytical models for the two categories were presented. Although the lens-type IRA has some advantages over the reflector-type IRA, a lens is generally heavier and more difficult to build than a reflector. Therefore, the reflector-type IRA was selected for further research.

Then, a numerical model for the reflector-type IRA was developed using the electromagnetic interactions generalized (EIGER) code suite. The IRA was efficiently modeled by removing redundancies in the geometry, such as the orthogonality and reflection symmetry. Because the geometry in the model was different from the original geometry, a method to obtain the response of the original geometry from the data generated by the model was presented. The performance of the numerical model was validated by comparison with an experimental model.

Next, the numerical model was used to obtain the responses of a number of IRAs with different geometries. These IRAs had different focal-length-to-diameter ratios \((F/D)\), reflectors, and TEM feed arm shapes. The characteristics of these IRAs were compared in terms of radiated waveforms, reflected voltages in the feeding transmission lines, etc. The length and height of the prepulse depended highly on \(F/D\). The postpulse could be lowered by increasing \(F/D\), using an offset reflector, and properly
terminating transverse-electromagnetic (TEM) feed arms. The reflected voltage in the feeding transmission line highly depended on the shape of TEM feed arms. Impulse height could be increased in the near field by using an elliptical reflector instead of a parabolic reflector.

For RVDs, an analytical model was developed using the thin wire approximation. The model was used to predict the radiated fields of an RVD loaded with the Wu-King profile. The model could also be used for RVDs in which the currents propagate at a speed slower than the speed of light in free space, which happens in insulated RVDs and RVDs printed on circuit boards.

Finally, an RVD was designed and realized. The RVD was printed on a circuit board and loaded with off-the-shelf surface-mount chip resistors according to the Wu-King profile. The realized RVD was shown to perform well.
CHAPTER I

INTRODUCTION

The radiation of a short electromagnetic pulse is useful in a variety of applications, such as non-destructive testing, ultra-wide band weapons, and remote sensing [1–4]. In remote sensing applications, short-pulse radiation is used to locate and identify targets. Typically, a short pulse is sent in the direction of a target, and the temporal information in the reflected signal is used to distinguish the target from other objects in the vicinity.

In order to radiate a short pulse, the antenna must be both nondispersive and capable of radiating over a broad bandwidth. Some broadband antennas suffer from severe dispersion. For example, when a conical spiral antenna is excited by a differentiated Gaussian pulse, the radiated waveform is a chirp [5]. For remote sensing applications, dispersion is an undesired characteristic because it will cause the signal from a target to be spread out in time making it more difficult to distinguish the target.

Impulse-radiating antennas (IRAs) and resistive vee dipoles (RVDs) are two of the antennas that can transmit and receive short pulses. The IRA is an equi-phase aperture that is fed by a spherical transverse electromagnetic (TEM) wave. A typical geometry is schematically shown in Figure 1.1. It consists of transverse electromagnetic (TEM) feed arms and a reflector. The TEM feed arms guide the outwardly propagating spherical TEM wave, which is launched at the apex of the TEM feed arms. The reflector converts the spherical wave into an equi-phase aperture field, which can also be done by a lens.

The IRA was introduced by Baum et al. [6], and a closed-form time-domain model
for predicting far fields of the IRA has been developed [7,8]. The closed-form model predicts that the radiated waveform from the IRA closely resembles the derivative of the excitation, e.g., it radiates a short Gaussian-like pulse when excited by a step-like pulse. Due to this pulse radiating capability, there has been great interest in applying the IRA to the near-field sensing applications [9–11].

A RVD is a dipole bent at an angle with resistively loaded arms. The purpose of the resistive loading is to obtain only an outwardly traveling wave along the arm without end reflections. In 1965, Wu and King showed that this can be achieved by loading a monopole with the “Wu-King” profile [12]. Since then, various works have discussed the transmission and reception characteristics of the monopole loaded with the Wu-King profile [13–18]. Recently, the RVD has been studied extensively for use in GPR applications [19–21].

The next two sections of this chapter will discuss the method of exciting the antennas, and introduce the software used to model the antennas. The last section of this chapter will provide an overview of this dissertation.
1.1 Antenna Excitation

Most of the analysis in this dissertation is conducted in the time domain using an input pulse incident in the feeding transmission line to the antenna. The input pulses considered in this paper are step-like, Gaussian, and differentiated Gaussian. The waveforms of these pulses and their frequency spectra are shown in Figure 1.2.

The step-like pulse is of the form

\[ V(t) = V_0 \left\{ \frac{1}{2} + \frac{1}{2} \text{erf} \left( k_1 \frac{t}{t_{10-90\%}} \right) \right\}, \] (1.1)

where \( t_{10-90\%} \) is the 10\% – 90\% rise time of the step-like pulse, \( \text{erf}(t) \) is the error function [22], and

\[ k_1 = 2 \text{erf}^{-1}(0.8) \simeq 1.8124. \] (1.2)

The step-like pulse has an infinite amount of energy at zero frequency (Figure 1.2 (b)), which does not radiate. However, the step-like pulse is frequently considered as an input to an IRA because the radiated waveform of the IRA for the step-like pulse closely resembles a Gaussian pulse, whose simple shape makes it easy to identify the reflections from a target.

The Gaussian function is of the form

\[ V(t) = V_0 e^{-\ln 16 (t/t_{FWHM})^2}, \] (1.3)

where \( t_{FWHM} \) is the full-width half-maximum of the Gaussian pulse. The 3dB frequency is related to \( t_{FWHM} \) as

\[ f_{1/2} = \frac{\ln 4}{\pi t_{FWHM}}. \] (1.4)

The differentiated Gaussian pulse is of the form

\[ V(t) = V_0 \frac{t}{t_{p-p}} e^{0.5 - 2(t/t_{p-p})^2}, \] (1.5)
Figure 1.2: Input pulses. Those in the left column are (a) step-like pulse, (c) Gaussian pulse, and (e) differentiated Gaussian pulse as functions of time. Those in the right column are the corresponding frequency spectrums of (b) step-like pulse, (d) Gaussian pulse, and (f) differentiated pulse.
where $t_{P,P}$ is the peak-to-peak interval of the differentiated Gaussian pulse. The peak frequency is related to $t_{P,P}$ as

$$f_{PK} = \frac{1}{\pi t_{P,P}}.$$  \hfill (1.6)

1.2 Numerical Modeling Software

Numerical models for the antennas have been developed, and the models have been used to investigate various characteristics of the antennas. The numerical models have been developed using the method of moments as implemented in the electromagnetic interactions generalized (EIGER) code suite\(^1\) [23, 24].

EIGER is an electromagnetic modeling software suite whose goal is to integrate a variety of frequency domain analysis methods, such as the method of moments and the finite element method, into a single software tool set [25]. The time-domain response of the antenna is obtained by multiplying the transfer function by an input pulse spectrum and transforming the product into the time domain. This is essentially the same as convolving the impulse response of the antenna with the input pulse in the time domain.

EIGER suite is composed of four elements: EIGER Build, EIGER Solve, EIGER Analyze, and EIGER Visual. The work flow of these elements is shown schematically in Figure 1.3. EIGER Build is a graphical tool that provides a visual validation of a geometry mesh and associates computational electromagnetic properties to the mesh. On Microsoft\(^\circledR\) Windows\(^\circledR\) or MS-DOS\(^\circledR\) operating systems, EIGER ANTS does what EIGER Build does on other platforms. EIGER Solve and EIGER Analyze are physics solvers. EIGER Solve calculates the primary quantities (e.g., mesh currents) and optionally some secondary quantities (e.g., fields). EIGER Analyze can also

\(^1\)The software is currently under development by multi-institutional collaboration. The latest version available is Version 1.0, beta release 2.2.
Figure 1.3: Work flow of EIGER software tool set.
calculate the secondary quantities using the primary quantities produced by EIGER Solve. EIGER Visual is a post processor that graphically displays the outputs of the physics solvers. EIGER Visual is currently available only on SGI® platforms [23, 25]. In preparing this dissertation, a MATLAB® program was written and used to visualize the physical quantities instead of EIGER Visual because MATLAB is more flexible and readily available on most popular platforms.

EIGER Solve and EIGER Analyze can be executed in parallel to increase the computational power using the message passing interface (MPI) standards. Most of the numerical results in this dissertation are calculated by running EIGER Solve and/or EIGER Analyze in parallel using MPI on the Beowulf cluster at the Electromagnetics/Acoustics Laboratory at the Georgia Institute of Technology.

1.3 Outline

The purpose of this dissertation is to investigate IRAs and RVDs using analytical, numerical, and experimental models.

In Chapter 2, IRAs will be categorized into two classes according to the method of forming an equi-phase aperture. Analytical models will be studied for each class of IRA. The parameters of the analytical models will be related to the parameters of the TEM feed structures.

In Chapter 3, the performance of the numerical model developed using EIGER will be validated by comparison with an experimental model. In the experimental model, a small dipole probe will be used to measure the radiated fields, and the measured fields will be compared to those obtained numerically. The reflected voltage in the feeding transmission line will also be measured and compared.

In Chapter 4, the numerical model will be used to analyze the characteristics of IRAs due to their geometrical variations. The geometrical variations will include two
focal-length-to-diameter ratios ($F/D$’s), an offset reflector, three ellipsoidal reflector, and a number of TEM feed arm terminations. The analysis will include the near- and far-field waveforms, power budgets, reflected voltages in the feeding transmission lines, etc.

In Chapter 5, RVDs with Wu-King profile will be studied. The radiated fields from a RVD with Wu-King profile will be investigated using a simple analytical model. The analytical model can predict the radiated fields of an insulated RVD as well as a RVD without insulation.

In Chapter 6, a discretely loaded RVD will be designed and realized on a printed circuit board (PCB). The performance of the realized RVD will be compared with that of the numerical model.

Chapter 7 will contain the summary and conclusion. Appendix A will discuss a method of calibrating the numerical model for a dipole sensor.
CHAPTER II

IMPULSE-RADIATING ANTENNAS

2.1 Introduction

An IRA is a focused aperture that is fed by a spherical TEM wave. Typically, the spherical TEM wave is guided by a pair of TEM feed arms\(^1\), which could be a TEM horn, bent circular cones, coplanar conical plates, etc [26, 27]. The spherical TEM wave is then converted to an equi-phase aperture by a lens (lens-type IRA) or a reflector (reflector-type IRA).

When an IRA is fed by a fast-rising step-like pulse, it radiates a Gaussian-like pulse that is short in time and directive in space [28]. The antenna can also be efficient because it can be made to radiate a large portion of the input energy by matching the characteristic impedance of the feeding transmission line to the TEM feed arms and appropriately terminating the TEM feed arms.

Baum et al. first introduced the IRA [6], and also presented simple closed-form time domain models to describe the radiated field of the IRA [7,29]. The models ignore multiple internal reflections and cannot describe the late-time waveforms; however, they are simple and accurate for the early time portions of the waveforms. In the next sections, the simple models are briefly studied.

\(^1\)Two types of feeds are frequently mentioned in this dissertation. One is TEM feed arms (or TEM feed structure), and the other one is feeding transmission line. The former is a portion of an antenna structure, which guides a spherical TEM wave to a reflector or a lens of an IRA. The latter is a balanced transmission line, which conveys the input pulses to an antenna.
2.2 Reflector-Type Impulse-Radiating Antennas

Figure 2.1 shows the typical structure of a reflector-type IRA. It consists of a parabolic reflector and TEM feed arms, which are connected to the reflector through low-frequency matching circuits. The characteristic impedance of the feeding transmission line is chosen to be the same as the characteristic impedance of the TEM feed arms ($Z_0$). The impedance of each low-frequency matching circuit is chosen to be $Z_0/2$ so that the TEM feed arms are terminated with a matched load at low frequencies. For this work, it is assumed that the feeding transmission line does not disturb the field generated by the antenna. Some configurations for feeding transmission lines are found in [30].

For the geometry shown in Figure 2.1, the radiated field on boresight is

$$\bar{E}_r(t_r) = \frac{V_0}{r} \frac{\bar{h}_a}{2\pi c f_g} \left\{ \frac{c}{2F} [-u(t_r) + u(t_r - 2F/c)] + \delta(t_r - 2F/c) \right\}, \quad (2.1)$$

when the antenna is excited at the apex of the TEM feed arms by an ideal step pulse with amplitude $V_0$, where $c$ is the speed of light in free space, $t_r = t - r/c$ is the retarded time, $\bar{h}_a$ is the aperture height, $f_g$ is the geometric impedance factor, $F$ is the focal length of the reflector, $u(t)$ is the Heaviside step function, and $\delta(t)$ is the Dirac delta function. Eq. (2.1) may be re-written for general time dependence:

$$\bar{E}_r(t_r) = \frac{\bar{h}_a}{2\pi c f_g r} \left\{ \frac{c}{2F} [-V(t_r) + V(t_r - 2F/c)] + \frac{d}{dt} V(t_r - 2F/c) \right\}, \quad (2.2)$$

where $V(t)$ is the voltage applied at the apex of the TEM feed arms. The geometric impedance factor $f_g$ is related to the characteristic impedance of the TEM feed line $Z_0$ [31,32]:

$$f_g = \frac{Z_0}{\eta_0}, \quad (2.3)$$

where $\eta_0$ is the wave impedance of free space. The aperture height $\bar{h}_a$ is defined as

$$\bar{h}_a = \frac{f_g}{V_0} \int_{S_a} \bar{E}(\bar{r}')dS', \quad (2.4)$$
Figure 2.1: Typical structure of an IRA with a pair of bent circular cones. $\alpha$ is the interior angle of the cone, and $\beta$ is the angle between the rotational axis of the reflector and the axis of the bent circular cone.
which is the integration of the aperture field $\vec{E}(r')$ over the aperture $S_a$ when the voltage $V_0$ is placed across the two TEM feed arms. It is usually assumed that the aperture plane is perpendicular to the rotational axis of the parabolic reflector and passes through the apex of the TEM feed arms.

Eq. (2.1) predicts that for a step excitation, the radiated field on boresight has two parts. These are called the prepulse and the impulse in literature. The step response of a typical IRA is shown in Figure 2.2. The prepulse is a direct radiation off the TEM feed arms toward the observer. The impulse is the aperture radiation, which is formed by reflection of the spherical TEM wave. There are late-time responses due to multiple internal reflections, which are not predicted by the simple model. The late-time responses are called postpulses and generally not wanted.

Three types of geometries are often considered for the TEM feed arms of the reflector-type IRAs: bent circular cones, coplanar conical plates, and curved conical plates. The geometric impedance factors of these types of TEM feed arms are described by simple mathematical expressions, but the expression for the aperture heights can be relatively complicated.
The geometry with bent circular cones is shown in Figure 2.1. The geometric impedance factor of a pair of bent circular cones is [33]

\[ f_g = \frac{1}{\pi} \cosh^{-1} \left( \frac{\sin \beta}{\sin \alpha} \right), \]  

(2.5)

where \( \alpha \) is the half interior angle of a cone arm, and \( \beta \) is the angle between the rotational axis of a cone arm and the plane of symmetry. When the blockage caused by the TEM feed arms is ignored, the aperture height formed by the TEM feed arms is \( D/2 \) [32]. Thus, the radiated field on boresight becomes

\[ E_r(t_r) = \frac{D}{4cr \cosh^{-1} \left( \frac{\sin \beta}{\sin \alpha} \right)} \left\{ \frac{c}{2F} \left[ -V(t_r) + V(t_r - 2F/c) \right] + \frac{d}{dt} V(t_r - 2F/c) \right\} \]  

(2.6)

for an input whose time dependence is \( V(t) \).

The geometry with the coplanar conical plates is shown in Figure 2.3. It is an appealing geometry because the TEM feed arms do not take space when the geometry is projected onto the aperture plane, and therefore they do not seem to block the aperture in the geometric optics sense, although the TEM feed arms do disturb electromagnetic fields and cause postpulses. The geometric impedance factor of a pair of the coplanar conical plates is [33]

\[ f_g = \frac{K(m)}{K(1-m)} \cdot m = \frac{\tan^2(\beta_1/2)}{\tan^2(\beta_2/2)}, \]  

(2.7)

where \( K(m) \) is the complete elliptic integral of the first kind [22], and \( \beta_1, \beta_2 \) are the angles between the inner and outer edges of a feed arm and the plane of symmetry.

The aperture height formed by the TEM feed arms has been calculated in [26] as

\[ h_a = \frac{\pi m^{-1/4}D}{4K(1-m)} \left[ 1 - \frac{2}{\pi} \sin^{-1} \left( \frac{(1-m^{1/2})^2}{1-m} \right) \right]. \]  

(2.8)

The first term of Eq. (2.8) is the aperture height when the blockage due to the TEM feed arms is ignored. The second term of Eq. (2.8) is the correction for the blockage.

Note that the geometry of the IRA with coplanar conical plates has a reflection symmetry. Because the electric fields are zero in the symmetry plane, a second pair of
Figure 2.3: Typical structure of an IRA with a pair of coplanar conical plates. \( \beta_1 \) and \( \beta_2 \) are the angles of the inner and outer edges of the TEM feed arms measured from the rotational axis of the reflector.

coplanar conical plates can be placed in the symmetry plane without disturbing the fields. The second pair of coplanar conical plates can be either driven independently or connected in parallel to the first pair. The former is useful for transmitting two polarizations. The latter is often preferred because it lowers the input impedance of the IRA by a factor of 2, which makes it easier to match the impedance of the antenna to the source [34].

The geometry with the curved plates is shown in Figure 2.4. The geometric impedance factor of a pair of the curved conical plates is [26]

\[
 f_g = \frac{K(m)}{K(1-m)}, \quad m = \left[\frac{1 - \sin \alpha}{\cos \alpha}\right]^4, \quad (2.9)
\]

where \( \alpha \) is the half angle of the fan that is formed by the projection of the TEM feed arms onto the aperture plane. The aperture height formed by this structure is [27]

\[
 h_a = \frac{\pi D}{2K(1-m)(1+m^{1/2})}, \quad (2.10)
\]
Figure 2.4: Typical structure of an IRA with a pair of curved plates. $\alpha$ is the half angle of the fan formed by projection onto the aperture.

where the aperture blockage caused by the TEM feed arms is not included. The expression of the aperture height including the blockage can be found in [26].

2.3 Lens-Type Impulse-Radiating Antennas

The lens-type IRA consists of TEM feed arms and a lens. A typical structure of the lens-type IRA is shown in Figure 2.5. Because there is no metal structure blocking the aperture, a lens-type IRA may have some advantages over a reflector-type IRA. However, a lens is generally more difficult to manufacture than a reflector, and it can also become very heavy when the aperture is large. Thus, the lens-type IRA is usually manufactured with a small aperture and used as a component of an array.

The simple model of a lens-type IRA for the radiated field on boresight is [35]

$$
\vec{E}^r(t_r) = \frac{V_0}{r} \frac{\tilde{h}_a \tau}{2\pi c f_g} \left\{ \delta(t_r) + \frac{c}{2l} [-u(t_r) + u(t_r - 2l/c)] \right\},
$$

(2.11)

when the antenna is excited at the apex of the TEM feed arms by an ideal step pulse.
with amplitude $V_0$ where $l$ is the length of the TEM feed arms. Eq. (2.11) may also be expressed for general time dependence:

$$
\bar{E}_r(t_r) = \frac{\bar{h}_0\tau}{2\pi cf_g r} \left\{ \frac{d}{dt} V(t_r) + \frac{c}{2l} \left[ -V(t_r) + V(t_r - 2l/c) \right] \right\},
$$

(2.12)

where $V(t)$ is the voltage applied at the apex of the TEM feed arms, and $\tau$ is the transmission loss through the lens:

$$
\tau = \frac{4\sqrt{\epsilon_r}}{(1 + \sqrt{\epsilon_r})^2},
$$

(2.13)

where $\epsilon_r$ is the dielectric constant of the lens.

The step response on boresight of a lens-type IRA is shown in Figure 2.6. Unlike a reflector-type IRA, the impulse part appears first and the radiation off the TEM feed arms follows next. Note that because the lens-type IRA is essentially an open-circuited transmission line at low frequencies, the reflection from the open ends of the TEM feed arms can be a problem [3]. To lessen this problem, matched loads can be placed between the TEM feed arms and the feeding transmission line as shown in

**Figure 2.5:** Typical structure of a lens-type IRA with a pair of curved conical plates.
Figure 2.6: Radiated field on boresight of a typical lens-type IRA as a function of time for a Heaviside step pulse input. The waveform begins with the impulse followed by the radiation off the TEM feed arms.

Figure 2.7: Schematic description of a lens-type IRA. Two matched loads of $Z_0/2$ are placed in series with the feeding transmission line and the TEM feed arms.
Figure 2.8: Typical structure of a lens-type IRA with a pair of flat plates. The width of a plate and the separation between plates are represented by $2a$ and $2b$.

Figure 2.7 [29]. In this case, the amplitudes of the voltage on the TEM feed arms and the radiated electric field are reduced by a factor of 2.

For TEM feed arms of a lens-type IRA, flat plates (Figure 2.8) and curved plates are usually considered. The geometric impedance factor of curved plates was described in the previous section. The solution to the geometric impedance factor of a pair of flat plates is implicit, so a set of equations must be solved numerically. The mathematical equations can be found in [27]. Asymptotic expressions are also available in [36] for small or large $b/a$.

The aperture height may be considered in two ways: finite aperture and infinite aperture, according to whether there is a conducting wall outside the aperture. The aperture height formed by curved plates was described in the previous section, and it is applicable to both finite aperture and infinite aperture [27]. For a pair of flat plates, the aperture height for an infinite aperture is

$$h_a = b,$$ (2.14)

which is the separation of a plate from the plane of symmetry at the aperture plane. The expression of the aperture height for a finite aperture is implicit and can only be solved numerically. The formulation can be found in [27].
2.4 Summary

This chapter studied the analytical models of IRAs. The IRAs were classified as either reflector-type IRAs or lens-type IRAs according to the aperture forming mechanism. The radiated field of each type of IRA was described by a simple closed form expression in the time domain.

The closed form expression required two parameters: the geometric impedance factor and the aperture height. These parameters for frequently considered TEM feed arms were presented. For reflector-type IRAs, bent circular cones, coplanar conical plates, and curved conical plates were considered for TEM feed arms. For lens-type IRAs, curved conical plates and flat plates were considered for TEM feed arms. For each type of TEM feed arm pair, the geometric impedance factor and the aperture height were presented.

The simple models were obtained by ignoring multiple internal reflections and by assuming ideal terminations of the TEM feed arms. To further understand the radiation characteristics of the IRA, a numerical analysis is required.

Although the lens-type IRA may have certain advantages over the reflector-type IRA, it has many disadvantages, such as difficulty of manufacturing, heavy weight, and energy loss in the matched loads. Thus, in the following chapters, only the reflector-type IRAs will be investigated further.
In this chapter, a numerical model for an IRA is developed using EIGER. The model is made efficient by removing redundancies in the IRA geometry. The performance of the model is validated by comparing the numerical results from the model with measured data.

3.1 Model for an Impulse-Radiating Antenna

Let us consider the IRA shown in Figure 3.1, whose dimensions are $F/D = 0.25$, $L/D = 0.19$, $\beta_1 = 73.87^\circ$, and $\beta_2 = 90^\circ$. The IRA has two pairs of tapered conical-coplanar-plate arms placed perpendicular to each other. The characteristic impedance of a pair of TEM feed arms is 400$\Omega$; two pairs of arms excited in parallel provide 200$\Omega$ input impedance. After the TEM waveguide, the tails of the arms are linearly tapered to 200$\Omega$ chip resistors. The arms with positive $y$ coordinates are connected together and the arms with negative $y$ coordinates are connected together.

To improve the efficiency of the numerical model, geometrical redundancies are removed. Because the arms of the IRA are placed perpendicularly, the fields due to one pair of arms are orthogonal to the fields due to the other pair. Thus, only one pair of the arms is necessary for the model. In addition, reflection symmetries are found across the E-plane (PEC symmetry) and the H-plane (PMC symmetry). If one
can use both symmetries at the same time, one needs to discretize only a quarter of the reflector. However, since EIGER does not support a PMC symmetry [37], half of the reflector is modeled using the PEC symmetry only.

A MATLAB® program was written to generate the mesh for the IRA. Figure 3.2 shows the mesh used for the numerical model. Half of the reflector and one arm are meshed. Triangular elements are used to mesh the parabolic reflector and the TEM feed arms. Wire elements are used to mesh the matching circuits (chip resistors) and the drive points. Note that the mesh density is increased around the edges of the reflector and the TEM feed arms to represent fast varying currents better. The mesh is further refined on the TEM feed arms because these are the places where the current density is high. The mesh contains 8,933 elements.

The mesh is read into EIGER Build, and computational electromagnetic properties are associated with the mesh. All the elements in the mesh are assumed to be PECs. The chip resistor, which is used as a low frequency matching circuit, is
Figure 3.2: Mesh for the IRA Model. (a) Mesh and its image (PEC reflection symmetry). (b) Detailed view of the mesh around the TEM feed arm termination. (c) Detailed view of the mesh around the apex. Delta-gap elements are marked with dots.
modeled using a delta-gap lumped impedance. The mesh is excited at the apex using a delta-gap voltage source. The electric field integral equation (EFIE) with linear basis functions is used to solve for the mesh currents.

EIGER Build produces an input file to EIGER Solve. EIGER Solve is executed in parallel using MPI to produce mesh currents, input impedance, and fields. The model is used to calculate these quantities at 160 frequencies from 125MHz to 20GHz ($D/\lambda_{min} = 20.4$, where $\lambda_{min}$ is the free space wavelength at 20GHz) with 125MHz increments on the Beowulf cluster at the Electromagnetics/Acoustics Laboratory in the Georgia Institute of Technology. The run time is approximately 91.3 hours using 32 computer nodes that each has an AMD Athlon™ XP 2200+ processor.

3.2 Synthesis of Electromagnetic Quantities

Because the modeled geometry is different from the original geometry, the data generated by the numerical model cannot be directly used for the antenna analysis. The data must be processed to produce valid results according to the relationship between the simplified model and the original geometry.

To obtain the response of the 4-arm IRA from the 2-arm IRA, the location of the observer must be analyzed. Consider an observer shown in Figure 3.3. Because the two pairs of TEM feed arms are orthogonal, the contribution from each pair of the TEM feed arms can be separated independently. Thus, the 4-arm IRA can be divided into two 2-arm IRAs with the observer at the same location. Then, by turning the antennas with the observers by $\pm 45^\circ$, the antennas have the same geometry. The new locations obtained by turning the antennas correspond to the locations where the response must be calculated for the construction of the response of the 4-arm IRA. Thus, to obtain the field of the 4-arm IRA at $(x_0, y_0, z_0)$ in the rectangular coordinate system or $(r_0, \theta_0, \phi_0)$ in the spherical coordinate system, the field of the 2-arm IRA
Figure 3.3: Location of the observer for the 4-arm IRA and corresponding observer locations for the 2-arm IRA.
must be calculated at \((x_1, y_1, z_1)\) and \((x_2, y_2, z_2)\) in the rectangular coordinate system or \((r_1, \theta_1, \phi_1)\) and \((r_2, \theta_2, \phi_2)\) in the spherical coordinate system, which are found as:

\[
\begin{bmatrix}
x'_1 \\
y'_1 \\
z'_1
\end{bmatrix} = \begin{bmatrix}
\cos \frac{\pi}{4} & -\sin \frac{\pi}{4} & 0 \\
\sin \frac{\pi}{4} & \cos \frac{\pi}{4} & 0 \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
x_0 \\
y_0 \\
z_0
\end{bmatrix},
\]

\[
\begin{bmatrix}
x'_2 \\
y'_2 \\
z'_2
\end{bmatrix} = \begin{bmatrix}
\cos \frac{\pi}{4} & \sin \frac{\pi}{4} & 0 \\
-\sin \frac{\pi}{4} & \cos \frac{\pi}{4} & 0 \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
x_0 \\
y_0 \\
z_0
\end{bmatrix}.
\]

or

\[
(r_1, \theta_1, \phi_1) = (r_0, \theta_0, \phi_0 + 45^\circ)
\]

\[
(r_2, \theta_2, \phi_2) = (r_0, \theta_0, \phi_0 - 45^\circ)
\]

(3.2)

Once the fields, currents, and input impedance are calculated from the numerical model, these quantities are synthesized for the original antenna. Because of the PEC symmetry used in the numerical model, the far fields \((E^r(\theta, \phi))\), near fields \((E(x, y, z))\), current \((I)\), and input impedance \((Z_{in})\) for the 2-arm IRA are obtained from the numerical model as

\[
\bar{E}^r_2(\theta, \phi) = \frac{\bar{E}^r_{num}(\theta, \phi)}{2},
\]

\[
\bar{E}_2(x, y, z) = \frac{\bar{E}_{num}(x, y, z)}{2},
\]

\[
I_2 = 2I_{num},
\]

\[
Z_{in,2} = 2Z_{in,num},
\]

where the quantities with the subscript 2 are those of the 2-arm IRA, and the quantities with the subscript \textit{num} are those obtained from the numerical model. Because the 4-arm IRA is equivalent to the two 2-arm IRAs driven in parallel, the current and input impedance for the 4-arm IRA are obtained as

\[
I = I_2/2,
\]

\[
Z_{in} = Z_{in,2}/2,
\]

(3.4)
Figure 3.4: Synthesis of 4-arm IRA from the 2-arm IRA.

The calculation of the fields for the 4-arm IRA is basically the vector addition of the fields for the two 2-arm IRAs. First, the 2-arm IRAs are turned by ±45° (Figure 3.4). Because the field components are turned, they should be projected on the primary coordinate axes to get the field components in the primary axes. Then, the 2-arm IRAs are overlapped, and appropriate fields are added vectorially. Mathematically,

\[
\begin{bmatrix}
E_x(x_0, y_0, z_0) \\
E_y(x_0, y_0, z_0) \\
E_z(x_0, y_0, z_0)
\end{bmatrix} = \begin{bmatrix}
\cos \frac{\pi}{4} & \sin \frac{\pi}{4} & 0 \\
-\sin \frac{\pi}{4} & \cos \frac{\pi}{4} & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
E_{x2}(x'_1, y'_1, z'_1) \\
E_{y2}(x'_1, y'_1, z'_1) \\
E_{z2}(x'_1, y'_1, z'_1)
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
\cos \frac{\pi}{4} & -\sin \frac{\pi}{4} & 0 \\
\sin \frac{\pi}{4} & \cos \frac{\pi}{4} & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
E_{x2}(x'_2, y'_2, z'_2) \\
E_{y2}(x'_2, y'_2, z'_2) \\
E_{z2}(x'_2, y'_2, z'_2)
\end{bmatrix},
\]

(3.5)
where $\vec{E} = E_x \hat{x} + E_y \hat{y} + E_z \hat{z}$ is the electric field for the 4-arm IRA, and $\vec{E}_2 = E_x^2 \hat{x} + E_y^2 \hat{y} + E_z^2 \hat{z}$ is the electric field for the 2-arm IRA.

Once the frequency domain results are available, they are transformed into the time domain with an input pulse incident in the feeding transmission line ($Z_0$). The reflected voltage in the feeding transmission line is obtained as

$$V_{refl}(t) = \mathcal{F}^{-1} \{ \Gamma(f)V_{in}(f) \},$$

$$\Gamma(f) = \frac{Z_{in}(f) - Z_0}{Z_{in}(f) + Z_0},$$

where $V_{in}(f)$ is the pulse incident in the feeding transmission line, and $\Gamma(f)$ is the reflection coefficient looking into the antenna from the feeding transmission line.

EIGER produces the electric fields as functions of the voltage at the drive point. Using these quantities, the electric fields as functions of the incident voltage in the feeding transmission line are obtained as,

$$\vec{E}(\vec{r}, t) = \mathcal{F}^{-1} \{ \vec{F}(\vec{r}, f)V_{drive}(f) \},$$

$$V_{drive}(f) = [1 + \Gamma(f)]V_{in}(f),$$

where $\vec{F}(\vec{r}, f)$ is the field due to the unit voltage at the drive point, and $V_{drive}(f)$ is the voltage at the drive point (i.e. the apex of the TEM feed arms).

### 3.3 Validation of the Model

#### 3.3.1 Impulse-Radiating Antenna Measurement

To validate the performance of the numerical model, the radiated fields and the reflected voltage in the feeding transmission line are measured and compared with those calculated by the numerical model. The measurement system (Figure 3.5) consists of two antennas: the IRA ($D = 30.6\text{cm}$) connected to port 1 of a network analyzer and a small dipole connected to port 2 of the network analyzer. For the electric field measurement, the small dipole is used as a field probe and samples the
Figure 3.5: Measurement setup for the radiated field of the IRA. (a) Measurement of the radiated fields ($S_{21}$). (b) Measurement of thru ($S'_{21}$) to compensate for the delay and attenuation along the baluns and coaxial cables. (c) Top view of the dipole probe. (d) Side view of the dipole probe.
fields at distances of 2.13m (7ft) to 7.62m (25ft) in 0.91m (3ft) increments from the apex of the IRA. At each distance, the scattering parameter $S_{21}$ is measured at 1601 equally spaced points over the frequency range of 63MHz to 10.143GHz (6.3MHz increments). The IRA and the dipole probe are fed by 100Ω balun assemblies that are connected to the network analyzer through coaxial cables.

The balun assembly is made using a Picosecond Pulse Labs Model 5315A Balun [38] that is connected to two semi-rigid 50Ω coaxial cables that are about 60cm long. The balun assembly is schematically shown in Figure 3.6. The coaxial cables are connected to the two output ports of the balun, which are approximately 180° different in phase. The outer conductors of these cables are connected together forming a 100Ω balanced transmission line. The 60cm-long cables provide a time window of about 5.8nsec in which no multiple reflections exist between the antenna and the balun.

To measure the electric field, the dipole probe is placed at a position, and the transmission coefficient ($S_{21}$) is taken by the network analyzer (Figure 3.5 (a)). This $S_{21}$ includes the delay and attenuation in the balun assembly. To compensate for the delay and the attenuation, the balun assemblies are connected together (Figure 3.5 (b)), and the transmission coefficient ($S'_{21}$) is taken by the network analyzer. This transmission coefficient ($S'_{21}$) is used to compensate $S_{21}$ for the delay and the
attenuation through the cable and balun assembly:

$$ T = \frac{S_{21}}{S_{21}'} . \quad (3.9) $$

The resultant $T$ is approximately the transmission coefficient from the IRA fed by the 100Ω feeding transmission line to the dipole probe connected to the 100Ω transmission line. The voltage in the 100Ω transmission line connected to the dipole probe is

$$ V_L(t) = \mathcal{F}^{-1} \{ T(f)V_{in}(f) \} . \quad (3.10) $$

The voltage waveform still includes the multiple reflections between the antenna and the balun and reflections from the surrounding objects, such as the equipment and the ground. These unwanted reflections are removed by gating in the time domain.

The reflected voltage in the feeding transmission line is also measured. First, the reflection coefficient $\Gamma_{ANA,IRA}$ is measured at port 1 of the network analyzer with the IRA in place (Figure 3.7 (a)). This measurement is equivalent to measuring the reflection coefficient ($\Gamma_l$) of the termination (IRA) that is connected to a two-port network (cable and balun assembly) [39]. The two-port network is characterized by a set of S-parameters ($S_{11}^B$, $S_{22}^B$, and $S_{12}^B S_{21}^B$); these parameters can be obtained by a series of measurements involving three standards (open, short, and match). The antenna side of the balun assembly is left open for the open ($\Gamma_l = 1$), shorted for the short ($\Gamma_l = -1$), and terminated with a 100Ω chip resistor for the match ($\Gamma_l = 0$). The diagram for this calibration procedure is shown in Figure 3.7.

The reflection coefficient measured at the network analyzer ($\Gamma_{ANA}$) is related to the S-parameters of the two-port network (cable and balun assembly) and its termination as:

$$ \Gamma_{ANA} = S_{11}^B + \frac{S_{12}^B S_{21}^B \Gamma_l}{1 - S_{22}^B \Gamma_l} . \quad (3.11) $$

The system of three equations resulting from the three standards is solved for the
Figure 3.7: Measurement setup for the reflected voltage in the feeding transmission line of the IRA. (a) Measurement of the reflection when the IRA is connected to the output of the balun assembly ($\Gamma_{ANA,IRA}$). (b) Measurement of the reflections with open, short, and matched load connected to the output of the balun assembly ($\Gamma_{ANA,open}$, $\Gamma_{ANA,short}$, and $\Gamma_{ANA,match}$).
three unknowns \((S^B_{11}, S^B_{22}, \text{ and } S^B_{12}S^B_{21})\):

\[
S^B_{11} = \Gamma_{ANA,\text{match}},
\]

\[
S^B_{22} = \frac{2S^B_{11} - \Gamma_{ANA,\text{short}} - \Gamma_{ANA,\text{open}}}{\Gamma_{ANA,\text{short}} - \Gamma_{ANA,\text{open}}},
\]

\[
S^B_{12}S^B_{21} = (\Gamma_{ANA,\text{open}} - S^B_{11})(1 - S^B_{22}),
\]

where \(\Gamma_{ANA,\text{open}}, \Gamma_{ANA,\text{short}},\) and \(\Gamma_{ANA,\text{match}}\) are the reflection coefficients measured by the network analyzer when the balun assembly is terminated with open, short, and matched load, respectively.

Once the S-parameters are obtained, then, the reflection coefficient from the IRA \((\Gamma)\) is calculated as

\[
\Gamma = \frac{\Gamma_{ANA,\text{IRA}} - S^B_{11}}{(\Gamma_{ANA,\text{IRA}} - S^B_{11})S^B_{22} + S^B_{12}S^B_{21}}.
\]

The reflection coefficients are measured at 1601 equally spaced points over the frequency range of 50MHz to 20.05GHz with 12.5MHz increment. The reflected voltage in the time domain is given by

\[
V_{\text{refl}}(t) = \mathcal{F}^{-1}\left\{\Gamma(f)V_{\text{in}}(f)\right\}.
\]

### 3.3.2 Dipole Probe Model

The measured data is obtained by sampling the radiated fields with a dipole probe. To compare the results from the measurement and the model, one could combine both the IRA and the dipole into a mesh for each distance and calculate the mesh for the dipole load voltage. However, this would require that the entire model be recalculated for every location of the dipole, which would require significant computation time.

Another approach is to model the dipole probe and the IRA separately and combine the results in post processing. Using this approach, both the IRA mesh and the dipole probe mesh only need to be calculated once. Because the dipole mesh size can be made small, this approach reduces the computation time by a factor of 7 (7 locations of the dipole probe in the measurement). Note that this method assumes
Figure 3.8: Mesh for dipole probe Model. It consists of 12 rectangular cells and is loaded with a 100Ω delta-gap lumped resistor at the center in order to simulate the 100Ω transmission line.

The electric field of the IRA has little spatial variation over a small region around the dipole. Because the length of the dipole is about a tenth of the wavelength at the highest frequency (10.143GHz), this assumption is valid except when the dipole probe is placed very close to the IRA.

Figure 3.8 shows the dipole mesh. It has 12 rectangular elements and is loaded with a 100Ω lumped resistor at the center, which simulates the 100Ω transmission line. The thin wires of the dipole are modeled with a tape-like structure. The width of the tape is determined by the modeling guideline suggested for use with EIGER [24]:

\[ W = ae^{1.5}, \]

(3.17)

where \( W \) is the width of the tape, and \( a \) is the radius of the wire. The tape is assumed to be a PEC, and the EFIE is used with linear basis functions.

Once the mesh currents are known, the currents through the 100Ω lumped resistor are found. Then the effective height of the dipole (\( \bar{h}_d(f) \)) is obtained:

\[ \bar{h}_d(f) = \frac{I_L(f)R}{E_0} \hat{y}, \]

(3.18)

where \( I_L(f) \) is the current through the load with resistance \( R = 100Ω \) when the dipole is illuminated with a planewave with amplitude \( E_0 \). The load voltage is expressed as:

\[ V_L(f) = \bar{h}_d(f) \cdot E_{inc}(f), \]

(3.19)
where $\vec{E}_{inc}(f)$ is the electric field incident on the dipole. By combining Eq. (3.7), Eq. (3.8), and Eq. (3.18), the load voltage when the dipole is placed a distance $r$ away from the IRA can be obtained as

$$V_L(t) = \text{FFT}^{-1}\left\{\tilde{h}_d(f) \cdot F(\vec{r}, f) T(f)V_{in}(f)\right\}. \quad (3.20)$$

This load voltage in the numerical model simulates the voltage in the 100Ω transmission line connected to the dipole probe.

### 3.3.3 Validation of Numerical Model

The numerical model is validated by comparison with measured results. First, the radiated fields are compared in terms of the received voltage by the dipole probe. The voltage waveforms from the measurement and the model are compared in Figure 3.9 for the three types of input pulses defined in Eq. (1.1), Eq. (1.3), and Eq. (1.5). In the figures, the voltages across the terminals of the dipole probe are plotted and shifted according to the location of the dipole. The IRA is excited by three types of input pulses with $t_{10\% - 90\%} = t_{FWHM} = t_{P\% - P\%} = 0.15\tau_a$ incident though 100Ω feeding transmission line, where $\tau_a = D/c$ is the time required for light to travel across the reflector.

In Figure 3.10, the reflected voltages from the measurement and the model are compared in the time domain for the three types of input pulses with $t_{10\% - 90\%} = t_{FWHM} = t_{P\% - P\%} = 0.075\tau_a$. For these graphs, the IRA is fed by a 200Ω transmission line, which is matched to the antenna. Because the characteristic impedance of the balun assembly used in the measurement is 100Ω, the reflection coefficient for the 200Ω feeding transmission line needs to be recalculated. First, the input impedance of the antenna, which is independent of the feeding transmission line, is calculated using the reflection coefficient obtained in Eq. (3.15):

$$Z_{in,IRA} = \frac{1 + \Gamma}{1 - \Gamma}. \quad (3.21)$$
Figure 3.9: Voltages across the terminals of the dipole probe as functions of time. The dotted lines represent the measured voltages, and the solid lines represent the voltages calculated using the numerical model. The dipole probe is placed a distance $r$ away from the IRA. The input pulses are (a) step-like, (b) Gaussian, and (c) differentiated Gaussian with pulse parameters $t_{10-90\%} = t_{FWHM} = t_{P-P} = 0.15\tau_a$. 

$4 \times 10^3$
Figure 3.10: Reflected voltages in the 100Ω feeding transmission line as functions of time. The dotted lines represent the measured voltages, and the solid lines represent the voltages calculated using the numerical model. The input pulses are (a) step-like, (b) Gaussian, and (c) differentiated Gaussian with pulse parameters $t_{10-90\%} = t_{FWHM} = t_{P-P} = 0.075\tau_a$. 
Then, the reflection coefficient for the 200Ω feeding transmission line is obtained as

$$\Gamma_{200\Omega} = \frac{Z_{in,IRA} - Z_0}{Z_{in,IRA} + Z_0},$$

(3.22)

where \( Z_0 = 200\Omega \). And finally \( \Gamma_{200\Omega} \) is used to obtain the reflected voltage in the 200Ω feeding transmission line as a function of time.

Figure 3.9 and Figure 3.10 show that the performance of the numerical model is good. The calculated electric field waveforms are in good agreement with the measured ones for all distances, times, and input pulses. In Figure 3.9, there is a slight mismatch between the amplitudes of the numerical and measured waveforms. This is due to the discontinuity in the connection between the two baluns during the calibration. The discontinuity reduces \( S'_{21} \), which leads to a larger \( T \) (Eq. (3.9)) than it should be and therefore results in over compensation in the waveform.

In Figure 3.10 (a), the results are seen to differ over the time interval \( 0 < t < 0.5\tau_a \). The reason for this difference is that the numerical model is not accurately predicting the characteristic impedance of the TEM feed arms due to the singularities in the currents on the edges of the TEM feed arms. The reflected voltage predicted by the numerical model over the time interval \( 0 < t < 0.5\tau_a \) reduces to almost zero when the characteristic impedance of the feeding transmission line is 189Ω. This indicates that the characteristic impedance of the TEM feed arms in the numerical model is approximately 189Ω, which is underestimated by approximately 5.5%.

In Figure 3.10 (a) – (c), the results are seen to differ near \( t \approx \tau_a \). This is believed to be caused by the small geometrical differences between the numerical and experimental models in the region near the terminating resistor.

As shown in Figure 3.9, these disagreements in the reflected voltages do not seem to degrade the performance of the numerical model in predicting the radiated fields.
3.4 Summary and Conclusions

A numerical model using EIGER was developed for an IRA. The IRA modeled in this chapter had two pairs of TEM feed arms placed orthogonal to each other. The IRA was modeled efficiently by eliminating geometrical redundancies. Only half of the reflector and one TEM feed arm were included in the numerical model.

Because the geometry of the numerical model was different from the original geometry, the results from the numerical model required further processing to obtain the fields of the original geometry. Details of this processing were presented.

To validate the performance of the numerical model, the results from the numerical model were compared with those obtained from measurements. The measurement system consisted of an IRA and a small dipole probe. Radiated fields along the boresight of the IRA were measured by the dipole probe. The reflected voltage in the feeding transmission line was also measured.

The dipole probe used in the measurement was numerically modeled, and the results from the dipole probe model were convolved with the results from the IRA model. The convolved results were shown to be in good agreement with the measured results.
CHAPTER IV

NUMERICAL ANALYSIS OF
IMPULSE-RADIATING ANTENNAS

The simple model predicts that the IRA will radiate a prepulse that is a replica of the input pulse and an impulse that is a derivative of the input pulse. However, the simple model does not include the multiple internal reflections that are caused by the presence of the TEM feed structure in front of the aperture; these internal reflections cause postpulses.

In addition, in the simple model, the TEM feed arms are treated as if they are infinitely long. However, for practical IRAs, the TEM feed arms must be terminated at the reflector. The discontinuity at the termination can cause a distortion in the prepulse waveform, and the reflected signal from the termination causes postpulses.

Some quantities cannot be obtained from the simple model. They include the near fields, power budgets, and the reflected voltage in the feeding transmission line. It is even difficult with the simple model to investigate the effect caused by a geometrical variation in the ideal IRA geometry. All these may be obtained with the numerical model.

In this chapter, a number of IRAs will be numerically modeled and analyzed. Some structural variations will be applied to the typical geometry of the IRA. In Sec. 4.1, the characteristics of two IRAs with different focal length to diameter ratios ($F/D$’s) will be compared. The reflected voltages in the feeding transmission lines and the radiated fields will be analyzed to show how $F/D$ and other parameters affect the waveforms. The near field waveforms, spot sizes, impulse and prepulse heights,
and power budgets are compared.

In Sec. 4.2, the IRAs with elliptical reflectors will be analyzed. The elliptical reflectors will focus the radiated fields at a point near the antenna instead of a point infinitely far away from the antenna. The waveforms at the optical foci and the reflected voltages in the feeding transmission lines will be compared for four IRAs with different locations of optical foci. The waveform changes will be graphed as they propagate in the near field, and the effects on the impulse heights will be discussed.

In an attempt to reduce the aperture blockage caused by the presence of the TEM feed arms, an IRA with an offset reflector will be numerically modeled in Sec. 4.3. The radiated waveforms will be compared and the reduction in the postpulse amplitudes will be discussed. The radiation pattern of the offset IRA will be compared with that of the center-fed IRA.

In Sec. 4.4, the TEM feed arms will be terminated with different shapes of curves, which are described by one equation. The sharpness of the curve will be related to the amplitudes of the reflected voltage in the feeding transmission line and the radiated field on boresight.

### 4.1 Focal-Length-to-Diameter Ratio

Two IRAs will be analyzed in this section: one with a focal-length-to-diameter ratio \((F/D)\) of 0.25 and one with a \(F/D\) of 0.5. Various characteristics of the IRAs, such as far- and near-field waveforms, illuminated spot sizes, and power budget, will be analyzed.

#### 4.1.1 Model for an Impulse-Radiating Antenna with \(F/D = 0.5\)

Let us consider the IRAs shown in Figure 4.1, whose dimensions are summarized in Table 4.1. Each of the two IRAs (flat IRA and tall IRA) has two pairs of ta-
Figure 4.1: Comparison of the two IRA geometries. (a), (b) top view, (c), (d) side view, (e), (f) feed arm dimensions.
Table 4.1: Parameters defining geometries of the IRAs.

<table>
<thead>
<tr>
<th></th>
<th>$F/D$</th>
<th>$L/D$</th>
<th>$\beta_0$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flat IRA</td>
<td>0.25</td>
<td>0.19</td>
<td>73.87°</td>
<td>90°</td>
<td></td>
</tr>
<tr>
<td>Tall IRA</td>
<td>0.5</td>
<td>0.5</td>
<td>53.01°</td>
<td>46.88°</td>
<td>59.94°</td>
</tr>
</tbody>
</table>

pered conical-coplanar-plate arms placed perpendicularly. The region inside a sphere of radius $L$ centered at the apex is an undisturbed spherical TEM waveguide. The angles associated with the spherical TEM waveguide are chosen such that the characteristic impedance is 400Ω; two pairs of arms excited in parallel provide 200Ω input impedance. After the TEM waveguide, the tails of the arms are linearly tapered to 200Ω chip resistors, which are used as low frequency matching circuits.

Although each pair of the arms can be excited independently, in this work, the arms with positive $y$ coordinates are connected together and the arms with negative $y$ coordinates are connected together [30]; thus, $y$-$z$ plane becomes the E-plane, and $x$-$z$ plane becomes the H-plane. The IRAs are fed by 200Ω transmission lines at the apexes of the TEM feed arms.

The flat IRA has already been modeled in Chapter 3. The tall IRA is modeled using the same procedure as used for the flat IRA, which involves removing geometrical redundancies using symmetry and orthogonality of the TEM feed arms. The response of the tall IRA, which has four arms, is obtained from the numerical model in the way shown in Chapter 3.

Figure 4.2 shows the meshes used for the numerical model. The meshes for the flat IRA and the tall IRA are shown for comparison. Triangular elements are used to mesh the paraboloidal reflectors and the arms. Wire elements are used to mesh the matching circuits and the drive points. The mesh for the IRA with $F/D = 0.5$ (tall IRA) contains 9,255 elements.

All the elements in the mesh are assumed PECs, and the chip resistor, which is used as a low frequency matching circuit, is included using a delta-gap lumped
Figure 4.2: (a), (b) Meshes used for the model of the IRAs. (c) Detail view of the mesh around the apex of the tall IRA. (d) Detail view of the mesh around the arm termination of the tall IRA. The location of the delta-gap elements are marked with dots.
impedance model. The mesh is excited at the apex by a delta-gap voltage source. The EFIE with linear basis functions is used to solve for the mesh currents. The model was used to calculate the response of the antenna at 160 frequencies from 125MHz to 20GHz \((D/\lambda_{\text{min}} = 20.4, \text{where } \lambda_{\text{min}} \text{ is the free space wavelength at } 20\text{GHz})\) with 125MHz increments on the Beowulf cluster at the Electromagnetics/Acoustics Laboratory in the Georgia Institute of Technology. The run time was approximately 99.2 hours for the tall IRA using 32 computer nodes, each one having an AMD Athlon™ XP 2200+ processor.

4.1.2 Analysis of the Impulse-Radiating Antennas

The simple model for the geometries shown in Figure 4.1 is described by Eq. (2.2) with Eq. (2.8). Figure 4.3 shows the boresight waveforms predicted by the simple model on top of the results from the numerical model. The waveforms are similar. However, there are differences in the prepulse waveform near the impulse, in the impulse height, and in the waveforms following the impulse. These differences result from the non-ideal TEM feed structure and the termination of the feed arms.

The radiation characteristics of the flat IRA and the tall IRA are viewed in the frequency domain in Figure 4.4. In Figure 4.4 (a), (b), the amplitude of the transfer function \((|T_E| = |E^r/V_{in}|)\) of the radiated field for the input pulse in the transmission line is shown as a function of frequency for the flat IRA and the tall IRA, respectively. Note that \(|T_E|\) is roughly proportional to \(D/\lambda\) because the aperture height of the IRA (Eq. (2.8)) is proportional to \(D\). The amplitude spectra of the ideal step function input are plotted in the same graphs.

The amplitude spectrum of the radiated field, \(|T_E||V_{in}|\), is plotted as a function of frequency for for the ideal step function input for the flat IRA and the tall IRA in Figure 4.4 (c), (d), respectively. In Figure 4.4 (c), (d), the spectra of the radiated fields are zero at \(D/\lambda = 0\) because the zero frequency components do not radiate. At
Figure 4.3: Boresight waveforms predicted by the numerical model (solid line) and the simple model (dotted line) when the input pulse is a step-like pulse with $t_{10-90\%} = 0.075\tau_a$. (a) F/D=0.25 and (b) F/D=0.5.
Figure 4.4: Radiated fields in the frequency domain. (a), (b) Transfer functions ($T_E$) describing the relations between the input pulses and the radiated fields and spectrum of the ideal step input $|V_{in}|$. (c), (d) Spectra of the radiated fields for the ideal step pulse.
frequencies other than zero, the spectrum for the radiated field is relatively constant. This is because $|T_E|$ is roughly proportional to $D/\lambda$ and $|V_n|$ is inversely proportional to $D/\lambda$. The constant spectrum is due to the impulsive part of the waveform, which is a delta function when the input is the ideal step function. The ripples in the frequency domain are due to the other parts of the waveform in the time domain, where the largest part is the prepulse.

Next, the reflected voltage in the transmission line and the radiated waveform on boresight are analyzed to see how each part of the antenna affects the waveform. Figure 4.5 shows the reflected voltages in the transmission line when the IRAs are fed by a step-like and a Gaussian pulse with the parameters $t_{10-90\%} = 0.075\tau_a$ and $t_{FWHM} = 0.075\tau_a$, respectively. In the figures, the first signal $A$ centered at $t/\tau_a = 0$ is the reflection from the terminals of the IRA. The reflection at $A$ is due to the approximation of the apex geometry (Figure 4.2 (c)).

The signal from the end of the TEM waveguide is marked with a $T$. Because of the taper in the arm tail, the wave on the arm experiences the gradually increasing impedance; thus, the wave is continuously reflected (positive) until the wave reaches the resistor. When the wave reaches the resistor, the wave experiences a sudden drop in the impedance (the 200$\Omega$ resistor and the reflector), and the wave is negatively reflected ($E$).

Meanwhile, the wave propagating toward the reflector is negatively reflected at the paraboloidal reflector. The reflected wave induces currents along the arms. $D$ and $P$ show two of the signal paths. Note that $D$ is observed first and $P$ is observed last because of the difference in the signal path lengths.

Figure 4.6 shows the radiated electric fields of the IRAs for the same input pulses as used in Figure 4.5. In the figures, $A$ marks the beginning of the direct radiation from the arms (prepulse). At the end of the TEM waveguide, the arms begin to radiate ($T$) because of the discontinuity due to the taper. At the end of the arms,
Figure 4.5: Analysis of the reflected voltages in the transmission line (a), (b) when the input pulse is a step-like with $t_{10-90\%} = 0.075\tau_a$, and (c), (d) when the input pulse is a Gaussian with $t_{FWHM} = 0.075\tau_a$. (e), (f) Wave and current paths.
Figure 4.6: Analysis of the radiated waveforms. (a), (b) Radiated waveforms when the input pulse is a step-like with $t_{10-90\%} = 0.075\tau_a$. (c), (d) Radiated waveforms when the input pulse is Gaussian with $t_{FWHM} = 0.075\tau_a$. (e), (f) Wave and current paths.
because of the sudden drop in the impedance, the wave is reflected and it is re-radiated at the apex (E).

The impulse $D$ is the reflected wave from the paraboloidal reflector. Some of the reflected wave is picked up by the arms, especially at the discontinuity, and re-radiated at the apex (P). Note that $D$ also includes the radiation from the end of the arms. They occur exactly at the same time. For the two IRAs, the impulse ($D$) heights seem to be roughly the same, but the prepulse ($A$) heights are higher for the flat IRA than for the tall IRA because of the directivity of the arm structure.

The reflected voltages in the transmission line are graphed as a function of time for a range of input pulse parameters in Figure 4.7. In both IRAs, the signals near $t = 0$ become more distinct with decreasing pulse parameter. However, note the interesting behavior of the waveforms of the tall IRA near $t = 1.3\tau_a$. In Figure 4.7 (c), the amplitude of the waveform increases with decreasing pulse parameter, $t_{10-90\%}$, but in Figure 4.7 (f), (i), the amplitude of the waveform decreases with decreasing pulse parameters in the ranges of $t_{FWHM} < 0.3\tau_a$ and $t_{P-P} < 0.3\tau_a$. The reason for this behavior is that, in the low frequency range, the reflection coefficient of the tall IRA results in the maximum response in the time domain for the pulses with $t_{FWHM} \simeq 0.3\tau_a$ and $t_{P-P} \simeq 0.3\tau_a$.

This is illustrated with a differentiated Gaussian pulse in Figure 4.8. The figure shows the the reflection coefficient of the tall IRA along with the frequency spectra of differentiated Gaussian pulses with a number of pulse parameters. The reflection coefficient has a peak at $D/\lambda \simeq 1.25$. Thus, input pulses with large frequency content near $D/\lambda \simeq 1.25$ will generate larger reflections. Thus, the input pulse with $t_{P-P} \simeq 0.3\tau_a$ will generate larger reflections than input pulses with a smaller or a larger pulse parameter.

The radiated electric fields on boresight are graphed as functions of time for a range of input pulse parameters in Figure 4.9. For long pulses, the impulse is not
Figure 4.7: Reflected voltages in the transmission line as functions of time for a range of input pulse parameters. (a), (d), (g) Step-like, Gaussian, and differentiated Gaussian pulses. (b), (e), (h) Reflected voltages from the flat IRA for the input pulses. (c), (f), (i) Reflected voltages from the tall IRA for the input pulses. Note that the vertical scales, which are shown at the top, are different for the input pulse waveforms.
Figure 4.8: Reflection coefficient of the tall IRA seen by the 200Ω transmission line. The frequency spectra of the differentiated Gaussian pulses with \( t_{p,p} / \tau_a = 0.4, 0.3, 0.2, 0.1, \) and 0.075 are also shown for comparison.

Distinguishable from the prepulse because the arms are too short to independently radiate a prepulse. For short pulses, the wave on the arms begins to radiate independently forming a prepulse. Note that the impulse height increases as the input pulse becomes shorter because the impulse height is proportional to the differentiation of the input pulse [6]. The prepulse heights are about the same for short pulses because the prepulses are the direct spherical radiation from the TEM waveguides.

Figure 4.9 suggests that the ratio of the power contained in the impulse to the power contained in the prepulse increases with decreasing pulse rise time. Figure 4.10 shows this ratio as a function of the rise time of the step-like pulse. For both antennas, the impulse ratio decreases with increasing pulse rise time. The ratio is higher for the tall IRA than for the flat IRA over the entire pulse rise time. The reason for this is that the prepulse amplitude of the tall IRA is smaller than that of the flat IRA.

Figure 4.11 shows the off-boresight waveforms as functions of time at a range of observation angles. In the figures, upper angles represent the observation angles in
Figure 4.9: Radiated field waveforms on boresight as functions of time for a range of input pulse parameters. Radiated field waveforms for (a), (b) step-like pulses, (c), (d) Gaussian pulses, and (e), (f) differentiated Gaussian pulses.
Figure 4.10: Ratio of the impulse power to the prepulse power as a function of pulse parameter. The input pulse is the step-like pulse.

the E-plane and the lower angles represent those in the H-plane. As one can see, the shape of the impulse is distorted off-boresight and the height decreases quickly as the observation angle increases; however, the shape and height of the prepulse are relatively independent of the observation angle.

Summary graphs are presented in Figure 4.12 for a range of step-like pulses. Two sets of graphs are made for each IRA. In each graph, lines in the right-hand side represent those in the E-plane and lines in the left-hand side represent those in the H-plane.

In Figure 4.12 (a) and (b), the impulse heights are shown as functions of observation angles. The impulse heights for the two IRAs are almost the same for the fast-rising pulses; however, the impulse heights are higher for the tall IRA for the slow-rising pulses. This is due to the the influence of the prepulses. The prepulse is a replica of the input pulse until the impulse begins at \(2F/c\) later. For a slow-rising pulse, the prepulse is turned off before it reaches its maximum. Because the prepulse of the tall IRA is longer, it rises more toward its maximum and has a larger area.
Figure 4.11: Patterns of the radiated waveform from the flat IRA and the tall IRA at different observation angles for (a), (b) a step-like pulse with $t_{10-90\%} = 0.075\tau_a$, (c), (d) a Gaussian pulse with $t_{FWHM} = 0.075\tau_a$, and (e), (f) a differentiated Gaussian pulse with $t_{P-P} = 0.075\tau_a$. 
Figure 4.12: Impulse and prepulse heights as functions of observation angles in the far-field for a range of step-like pulses. (a), (b) Impulse heights. (c), (d) Prepulse heights.
than that of the flat IRA. Because the impulse area is the same as the prepulse area, a larger prepulse area results in a higher impulse [33].

The prepulse heights are shown in Figure 4.12 (c) and (d) as functions of observation angles. Ideally, the prepulse height is smaller on boresight than off boresight because of the directivity of the arm structure; if the arms were infinitely long without a reflector, the maximum would be at 180° and the minimum at 0°. However, the reflector interferes with the prepulse at angles off boresight and for slow-rising input pulses. As the rise time of the input pulse decreases, the prepulse is less affected by the reflector on boresight and the prepulse height converges to a value (Eq. (2.2)). The prepulse height of the tall IRA converges faster than that of the flat IRA because the arms of the tall IRA are longer than those of the flat IRA.

In some applications, the ratio of the impulse height to the prepulse height may be important. This ratio is graphed for the IRAs in Figure 4.13 (a), (b) for a range of step-like pulses. In each graph, lines in the right-hand side represent those in the E-plane and lines in the left-hand side represent those in the H-plane. This ratio is much higher in the tall IRA due to its low prepulse height.

In Figure 4.14, graphs of the normalized electric fields in the near field are shown. For each IRA, the normalized quantity \( rE_y/V_0 \) on the axis is graphed to show the effect of the observation distance when the IRA is fed by a step-like, Gaussian, and differentiated Gaussian with pulse parameters \( t_{10-90\%} = 0.075\tau_a \), \( t_{FWHM} = 0.075\tau_a \), and \( t_{P-P} = 0.075\tau_a \), respectively. The origin of the coordinate systems is taken to be the apexes of the IRAs. The impulse develops as the observer moves into the far-zone of the IRAs. The prepulse height stays the same because it originates at the apex and propagates spherically. Likewise, the heights of other re-radiations radiating from the apexes, such as \( \mathbf{E} \) and \( \mathbf{P} \) (see Figure 4.6), stay the same.

As shown in Figure 4.14, the impulse develops as the observer moves farther out. This is well understood [6,40,41]. A time domain illustration of the aperture radiation
Figure 4.13: Impulse height to prepulse height ratios as functions of observation angles in the far-field for a range of step-like pulses. (a) The ratios for the flat IRA ($F/D = 0.25$). (b) The ratios for the tall IRA ($F/D = 0.5$).
Figure 4.14: Graphs of the normalized electric fields ($rE_y/V_0$) on boresight as functions of time at different observation distances. The input pulses are (a), (b) step-like with $t_{10-90\%} = 0.075\tau_a$, (c), (d) Gaussian with $t_{FWHM} = 0.075\tau_a$, and (e), (f) differentiated Gaussian with $t_{P-P} = 0.075\tau_a$. 
in the near-zone can be found in [41].

Figure 4.15 shows the normalized impulse heights (maximum of $rE_y/V_0$) as functions of distance for step-like pulses. One can see that the impulse quickly grows and converges to the far-field height. Figure 4.15 (c), (d) show the ratios of the impulse heights to the prepulse heights as functions of observation distance up to $10D$. Because the height of the normalized prepulse (minimum of $rE_y/V_0$) stays the same for all distance, Figure 4.15 (c), (d) are exact replicas of Figure 4.15 (a), (b) for the observer distance up to $10D$, except the y-axes are scaled by the inverse of the prepulse height.

As shown in Figure 4.12, the impulses are strong on boresight and weak off boresight, implying the impulse beam width of the IRA. To quantify the impulse beam width in the near zone, graphs of the size of the illuminated area ($\Delta W$) on a plane normal to the axis at a distance of $r$ away from the IRAs are presented in Figure 4.16. The half-width half-maximum of the impulse heights (spot sizes) are plotted for step-like input pulses. The lines in the upper half represent the spot sizes in the E-plane and the lines in the lower half represent the spot sizes in the H-plane. The figure shows that the spot sizes are smaller for the flat IRA than they are for the tall IRA, and therefore the impulse beam width of the flat IRA is narrower than that of the tall IRA in the near zone. The figure also shows that the spot sizes are smaller in the H-plane than those are in the E-plane. The spot sizes decrease as the pulse rise time decreases.

In Figure 4.12 and Figure 4.15, it is seen that as the pulse rise time decreases, the amplitude of the impulse increases, implying that the antennas are more efficient at higher frequencies. Figure 4.17 shows the power budgets of the two IRAs as functions of normalized frequency. In the figure, the reflected power in the transmission line, the dissipated power in the matching circuits, and the radiated power are shown. The powers are normalized by the input power.
Figure 4.15: Impulse and prepulse heights as functions of observation distance in the near-field for a range of step-like pulses. (a), (b) Impulse heights. (c), (d) Impulse height to prepulse height ratios.
Figure 4.16: Spot sizes for a range of step-like pulses. In each graph, the lines in the upper half are those in the E-plane and the lines in the lower half are those in the H-plane.

For each IRA, the reflected power is small over the entire frequency range because the IRA is well matched to the 200Ω-transmission line. At low frequencies, the power dissipated in the matching circuits is larger than the power radiated. However, at high frequencies, the power radiated is larger than the power dissipated, and the radiated power increases as the frequency increases, reaching almost unity at the highest frequency. This indicates that the radiation efficiency of the IRA increases with decreasing pulse rise time.

Note the frequency where the radiated power and the dissipated power are equal. This frequency is lower for the tall IRA because it radiates better at low frequencies due to its more spread-out prepulse. It was noted in Figure 4.12 (c) and (d) that, for a slow-rising pulse, the amplitude of the radiated prepulse (low frequency content of radiation) depends on the prepulse width.
Figure 4.17: Power budgets of (a) the flat IRA and (b) the tall IRA as functions of normalized frequency. The powers are normalized by the input power.
4.1.3 Conclusion

A numerical model was developed for the IRA. Using the numerical model, various quantities were calculated for step-like, Gaussian, and differentiated Gaussian pulses with a range of pulse parameters from $0.075\tau_a$ to $0.4\tau_a$.

Waveforms of the reflected voltage in the transmission line and far- and near-zone electric fields were analyzed. Impulse heights as functions of the observation angle and distance were calculated. Spot sizes were calculated as functions of distance. Power budgets were analyzed in the normalized frequency range from 0 to $18D/\lambda$.

With a larger $F/D$, the prepulse became longer and lower as expected from the simple model. The radiation efficiency at low frequency was higher with a larger $F/D$ because the prepulse was longer. The postpulse amplitude was lower with a larger $F/D$.

4.2 Elliptical Reflector

When an elliptical reflector is illuminated by a source located at one of its two foci, the signal from each elementary position on the reflector arrives at the other focus with the same phase, and the geometric optics rays converge at the focus. Because of this property, the ellipsoidal reflector has been attractive to a number of applications such as illumination of biological specimens, power transfer between antennas in the near field, etc [42–45]. In these applications, the transmitting antenna is placed at or near one focus, and the transmitted energy is expected to converge at the other focus.

By combining this focusing characteristic of the ellipsoid and the ultra-wideband radiating characteristics of the impulse-radiating antenna [46], one may be able to increase the impulse height and/or reduce the illuminated impulse spot size in the near field. The focused impulses may make it easy to sense a target, for certain sensing
applications where the impulse part of the waveform is of considerable importance.

4.2.1 Modeling of the Impulse-Radiating Antennas

Consider a geometry shown in Figure 4.18 (a). It is an ellipsoid whose equation may be written as

$$\frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{b^2} = 1, \quad a < b,$$

(4.1)

where the major axis is assumed to lie along the z-axis. The distance between the foci, $\Phi$, and the distance between focus 1 and the vertex closest to the focus 1 can be written in terms of $a$ and $b$ as:

$$\Phi = 2\sqrt{b^2 - a^2},$$

$$F = b - \sqrt{b^2 - a^2}.\quad (4.2)$$

A dish reflector with diameter $D$ is shaped along a part of the ellipsoid. A pair of conical-coplanar-plate feed arms is attached to the reflector with the apex placed at focus 1 as shown in Figure 4.18 (b).

The angles associated with the feed arms are chosen such that the characteristic impedance is $Z_0 = 400\Omega$ [33]. Each feed arm is terminated by a 200Ω resistor as a low frequency matching circuit. After a radial distance $L$ from the apex, the arms are linearly tapered to the resistors. One can add a second pair of TEM feed arms perpendicular to the first one to lower the antenna input impedance by a factor of two or to achieve a dual polarization [30].

Three IRAs with $\Phi/D = 0.5, 1.0, \text{ and } 1.5$ are numerically modeled and compared with an IRA with a parabolic reflector, which corresponds to $\Phi/D = \infty$. The dimensions of the IRAs are summarized in Table 4.2. The differences of the geometries are seen in Figure 4.19, where the diagrams of the IRAs are drawn on top of each other.

Triangular meshes are generated for the antennas. To improve the efficiency of the numerical model, different meshes are used according to the frequency, and PEC symmetry is utilized noting the reflection symmetry across the $x$-$z$ plane. The high
Figure 4.18: Geometry of the focused IRA. (a) The focused IRA system and its foci. (b) The focused IRA and its dimensions.
Table 4.2: Parameters defining geometries of the IRAs.

<table>
<thead>
<tr>
<th>$\Phi/D$</th>
<th>$F/D$</th>
<th>$L/D$</th>
<th>$L_A/D$</th>
<th>$\beta_0$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$Z_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5732</td>
<td>60.72°</td>
<td>53.85°</td>
<td>68.08°</td>
<td>400Ω</td>
</tr>
<tr>
<td>1.0</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5918</td>
<td>57.67°</td>
<td>51.03°</td>
<td>64.82°</td>
<td>400Ω</td>
</tr>
<tr>
<td>1.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.6005</td>
<td>56.37°</td>
<td>49.85°</td>
<td>63.44°</td>
<td>400Ω</td>
</tr>
<tr>
<td>$\infty$</td>
<td>0.5</td>
<td>0.5</td>
<td>0.6250</td>
<td>53.13°</td>
<td>46.88°</td>
<td>59.94°</td>
<td>400Ω</td>
</tr>
</tbody>
</table>

Figure 4.19: Diagrams of the IRAs. The diagrams of the IRAs with $\Phi = 0.5D$, $1.0D$, and $1.5D$ are drawn on top of each other to show the differences in the geometries.

Frequency meshes contain 10658, 10425, and 10255 triangle elements, and the low frequency meshes contain 5233, 5117, and 4986 triangle elements for the IRAs with $\Phi/D = 0.5$, 1.0, and 1.5, respectively. An example mesh is shown in Figure 4.20. The mesh is for the IRA with $\Phi/D = 0.5$. The approximations of the apex and matching circuit geometries are shown in Figure 4.20 (b), (c).

The response of each antenna is obtained at 150 equally spaced points within the normalized frequency range of $D/\lambda = 0.102$ to $D/\lambda = 15.3$, where $\lambda$ is the free space wavelength. The lower 75 and the upper 75 frequencies are calculated using the low frequency model and the high frequency model, respectively. It takes approximately 79.0, 72.3, and 66.8 hours to produce mesh currents for the IRAs with $\Phi/D = 0.5$, 1.0, and 1.5, respectively, on the Beowulf cluster using 32 AMD Athlon™ XP 2200+ processors.
Figure 4.20: An example mesh for the focused IRA. (a) The mesh used for the numerical model and its image resulting from the PEC reflection symmetry. (b) The apex approximation. (c) The arm termination with a matching circuit. For this mesh, $\Phi/D = 0.5$. 
4.2.2 Analysis

In Figure 4.21 (a), (b), the reflected voltages in the feeding transmission line are shown as functions of the normalized time, $t/\tau_a$, where $\tau_a = D/c$ is the time for light to travel the length of the reflector diameter, $D$. The input pulses in the feeding transmission line are a step-like and a Gaussian with $t_{10-90%}/\tau_a = t_{FWHM}/\tau_a = 0.1$. Magnifications of the regions surrounded by the boxes are shown in Figure 4.21 (c), (d).

In the figure, the first pulse around $t/\tau_a = 0$ is the reflection due to the approximation of the apex used in the model (Figure 4.20 (b)). After the first pulse, the waveform is seen to have nonzero value over the time interval $0.2 < t/\tau_a < 0.8$, where there should be no reflection. This is due to a small error in the characteristic impedance of the TEM feed structure predicted by the model ($\sim 1\%$).

The pulse around $t/\tau_a = 1.2$ is the superposition of the signals from the reflector and the terminations of the TEM feed arms. The amplitude of the pulse increases with decreasing $\Phi$ because the signal from the reflector surface is more directive toward the apex of the TEM feed arms with decreasing $\Phi$.

Note that the maximum of the pulse occurs earlier as $\Phi$ decreases. The reason for this is that the signal from the reflector arrives earlier with smaller $\Phi$ because the TEM feed arms are shorter as shown in Figure 4.19. The pulse is also sharper with smaller $\Phi$ because the positive signal from the TEM feed arm termination is stronger due to sharper termination. The positive signal from sharper TEM feed arm termination eats more into the negative signal from the reflector.

In Figures 4.22 – 4.25, normalized fields at the four optical foci of the reflectors are graphed as functions of time for a step-like pulse with $t_{10-90%}/\tau_a = 0.1$. The impulses are magnified in (b), and the prepulses and postpulses are magnified in (c). Because $F/D = 0.5$, the round-trip time taken by the signal from the apex to reach the reflector and reflect back to the apex is $1.0\tau_a$, and therefore the signal from the
Figure 4.21: Voltage reflected into the feeding transmission line. Results are shown for (a) a step-like pulse and (b) a Gaussian pulse with $t_{10-90%}/\tau_a = t_{FWHM}/\tau_a = 0.1$. The regions surrounded by the boxes in (a), (b) are magnified in (c), (d).
Figure 4.22: Comparison of the electric fields at $r/D = 0.5$ for a step-like pulse with $t_{10-90%}/\tau_a = 0.1$. Magnifications of the signals are shown in (b) and (c).
Figure 4.23: Comparison of the electric fields at $r/D = 1.0$ for a step-like pulse with $t_{10-90%}/\tau_a = 0.1$. Magnifications of the signals are shown in (b) and (c).
Figure 4.24: Comparison of the electric fields at $r/D = 1.5$ for a step-like pulse with $t_{10-90%}/\tau_a = 0.1$. Magnifications of the signals are shown in (b) and (c).
Figure 4.25: Comparison of the electric fields at $r/D = \infty$ for a step-like pulse with $t_{10-90%}/\tau_a = 0.1$. Magnifications of the signals are shown in (b) and (c).
center of the reflector is retarded by the signal from the apex by \(1.0\tau_a\).

Note that in each figure, the impulse centered at \(t_r/\tau_a = 1.0\) is that from the IRA with \(\Phi = r\). The impulse from the IRA with \(\Phi > r\) appears later than \(t_r/\tau_a = 1.0\), and the impulse from the IRA with \(\Phi < r\) appears earlier than \(t_r/\tau_a = 1.0\). This is related to the signal path of the impulse for an observer on axis (e.g., apex – reflector – observer on axis). The length of this signal path varies as what part of the reflector is taken as the signal path. The signal path of the impulse is always \(2F + r\) for the observer at a distance \(r\) away from the antenna when the center of the reflector is taken as the signal path. However, the signal path of the impulse is shorter than \(2F + r\) for the observer at a distance \(r > \Phi\), longer than \(2F + r\) for the observer at a distance \(r < \Phi\), and equal to \(2F + r\) for the observer at a distance \(r = \Phi\) when other part of the reflector is taken as the signal path. Since the impulse is the superposition of the signal traveling each elementary path, the impulse from the IRA with \(\Phi = r\) is centered at \(t_r/\tau_a = (2F + r)/D - 2F/D = 1.0\) because all the signal paths have the same length. The impulse from the IRA with \(\Phi > r\) is centered at \(t_r/\tau_a = (2F + r)/D - 2F/D \leq 1.0\) because all the signal paths are shorter than or equal to \(2F + r\). The impulse from the IRA with \(\Phi < r\) is centered at \(t_r/\tau_a = (2F + r)/D - 2F/D \geq 1.0\) because all the signal paths are longer than or equal to \(2F + r\).

Due to the focusing, the amplitude of the impulse from the IRA with \(\Phi = r\) is expected to be higher than the impulses from the other IRAs; this is true in Figure 4.22, 4.23, and 4.25. However, it is not true in Figure 4.24. Because \(r/D\) is 1.5 for Figure 4.24, the impulse from the IRA with \(\Phi/D = 1.5\) is expected to be the largest. However, the impulse from the IRA with \(\Phi/D = 1.0\) is larger in amplitude than the impulse from the IRA with \(\Phi/D = 1.5\). This is because the aperture is not large enough for the optical model to be accurate; for an aperture of this size, the maximum amplitude will occur between the aperture and the optical focus [47,48]. If
the aperture is very large \((D/\lambda \gg 1)\), the maximum will occur at the optical focus.

The pulse around \(t_r/\tau_a = 1.2\) is the re-radiation, at the apex, of the signals from the reflector edge and the TEM feed arm termination. This pulse occurs earlier for the IRA with a smaller \(\Phi\) because the path length of this pulse depends on the TEM feed arm length, \(L_A\), which is smaller for the IRA with a smaller \(\Phi\). The postpulse near \(t_r/\tau_a = 2\) is the superposition of signals resulting from a number of mechanisms including internal reflections and the re-radiation of the reflected voltage shown in Figure 4.21. The amplitude of the postpulse is larger with a smaller \(\Phi\), and the normalized amplitude grows as the pulse propagates. This indicates that some part of the postpulse near \(t_r/\tau_a = 2\) is the signal from the reflector, which is formed by multiple internal reflections.

The prepulse amplitude is larger with a smaller \(\Phi\). Because the included angle of the TEM feed arms is larger, less energy is guided toward the reflector, and therefore more energy is directly radiated. The electric fields in the near zone have nonzero value at late times because the observer in the near zone sees the static field of the antenna resulting from the late-time amplitude of the step-like input pulse.

Figure 4.26 shows the development of the impulses in the vicinity of the antennas. The electric field on \(z\)-axis is plotted as a function of time \((t/\tau_a)\) for a step function with \(t_{10-90%}/\tau_a = 0.075\) and vertically offset by the distance from the antenna. For the antennas with a finite \(\Phi\), the electric field at focus 2 is plotted with a dotted line.

Note that the maximum impulse amplitude does not occur at focus 2. It rather appears closer to the antenna due to the finite size of the aperture. Note also that the impulse of the antenna with a smaller \(\Phi\) grows faster and also decays faster, and in the vicinity of the antenna, the impulse amplitude close to the antenna decreases with increasing \(\Phi\).

In Figure 4.27, impulse amplitude of each antenna is plotted as a function of distance for step-like functions with \(t_{10-90%}/\tau_a = 0.075\) and 0.2. The impulse amplitudes
Figure 4.26: Waterfall graphs of the normalized electric fields ($E_y/E_0$) for the IRAs with $\Phi/D = (a) 0.5$, (b) 1.0, (c) 1.5, and (d) $\infty$. Each line represents the electric field at a point on $z$-axis, which is plotted as a function of time ($t/\tau_a$) for a step function with $t_{10-90%}/\tau_a = 0.075$ and vertically offset by the distance from the antenna. The electric field observed at focus 2 of each antenna is plotted with a dotted line (except for (d)). The numbers on the lines are the normalized distance ($r/D$) for the lines.
Figure 4.27: Impulse amplitudes as functions of distance for step-like functions with $t_{10-90%}/\tau_a = 0.075$ and 0.2. The impulse amplitudes at distances close to the antennas are magnified in (b).
close to the antennas are magnified in (b).

The impulse amplitude is seen to depend strongly on \( t_{10-90\%} \). The impulse amplitude is larger with a smaller \( t_{10-90\%} \). This is expected because the radiation from a finite aperture depends on the derivative of the aperture field [41].

Note that as a result of focusing, the maximum impulse amplitude increases significantly, and the location of the maximum impulse occurs closer to the antenna with decreasing \( \Phi \) for a step-like pulse with \( t_{10-90\%}/\tau_a = 0.075 \). The impulse amplitude for the IRA with a finite \( \Phi \) is larger over a small range of distances \( (r/D < 1.7) \) for a step-like pulse with \( t_{10-90\%}/\tau_a = 0.075 \). However, for a step-like pulse with \( t_{10-90\%}/\tau_a = 0.2 \), the maximum impulse amplitude increases slightly with decreasing \( \Phi \), the location of the maximum impulse does not vary much as a function of \( \Phi \), and the impulse amplitude for the IRA with a finite \( \Phi \) is larger over a large range of distances \( (r/D < 10) \). Therefore, the focusing property appears more clearly for a step-like pulse with a smaller \( t_{10-90\%} \). This is because the aperture size is seen electrically larger for a step-like pulse with a smaller \( t_{10-90\%} \).

In Figure 4.28, the illuminated impulse spot sizes are graphed as functions of the normalized distance for step-like pulses with \( t_{10-90\%}/\tau_a = 0.075 \) and \( 0.2\tau_a \). For the graphs, an imaginary plane orthogonal to both the E-plane (\( y-z \) plane) and H-plane (\( x-z \) plane) is placed at a coordinate \( z \). Then, the location of the half maximum of the impulse amplitudes are plotted for each \( z \). The lines in the upper half of each graph are those in the E-plane and the lines in the lower half are those in the H-plane.

The spot sizes are smaller with a smaller \( \Phi \) at distances very close to the IRAs. However, at large distances, the spot sizes are larger with a smaller \( \Phi \) as if the impulses diverge after the foci. Note that the minimum waists are not at the foci. For the IRA with \( \Phi = \infty \), the minimum waist is around \( r/D = 0.55 \) for \( t_{10-90\%}/\tau_a = 0.075 \) and \( r/D = 0.7 \) for \( t_{10-90\%}/\tau_a = 0.2 \). Thus, one may not be able to get the minimum waist farther than those distances with finite \( \Phi \)’s for these input pulses.
Figure 4.28: Illuminated impulse spot sizes as functions of normalized distance. The lines in the upper half of each graph are those in the H-plane and the lines in the lower half are those in the E-plane. The input pulses are step-like pulses with (a) $t_{10-90%}/\tau_a = 0.075$ and (b) $t_{10-90%}/\tau_a = 0.2$. 
Although the minimum spots are not obtained at distances \( r/D = 1 \) or 2, the spot sizes are smaller there with \( \Phi/D = 1.0 \) or 1.5 than they are with \( \Phi = \infty \). In addition, the spot size decreases significantly at distances close to the antenna with decreasing \( \Phi \) for a step-like pulse with smaller \( t_{10-90\%} \). This indicates that the impulse is more directive toward a target in the near field with a finite \( \Phi \) for a step-like pulse with smaller \( t_{10-90\%} \). Therefore, an IRA with an elliptical reflector fed by a fast-rising step-like pulse can be useful in near-field sensing applications because it can have a smaller focused spot at distances close to the antenna.

### 4.2.3 Conclusion

The focused IRA has been numerically modeled and analyzed in the time domain. The focused IRA consisted of a reflector shaped along a part of an ellipsoid and a pair of conical-coplanar-plate feed arms with its apex placed at one of the two ellipsoid foci. The IRA was fed by a 400Ω transmission line, which is matched to the feed arms.

The reflected voltages in the feed transmission line and the electric field waveform at the other focus has been graphed and analyzed. The impulse heights were graphed for a range of foci separation (\( \Phi \)). The illuminated impulse spot sizes were graphed as functions of distance.

The maximum impulse height and the minimum spot size depend on the pulse rise time as well as \( \Phi \), and they do not occur at the optical focus. However, the spot sizes of the focused IRA (\( \Phi \)) are smaller at the distance very close to the antenna than the spot sizes of the unfocused IRA (\( \Phi = \infty \)). This feature may be used in the near field sensing applications where the impulse part of the radiated waveform is of considerable importance.
4.3 Offset Reflector

Typically, the apex of the TEM feed structure is placed on the rotational axis of the circular paraboloidal reflector. In this center-fed geometry, the existence of the TEM feed structure causes aperture blockage [49–51]. Although conical coplanar plates are widely used for a TEM feed structure [52–55] because they have no optical blockage, they still partially block the aperture because of the scattering from the TEM feed structure.

To further reduce the blockage, an offset reflector will be investigated in this section. The IRA with an offset reflector was suggested in [6, 56]. In this geometry, the apex of the feed structure is still located at the focal point of the reflector, but the feed structure is placed off the path of the reflected waves, resulting in essentially no blockage.

In this section, two offset IRAs are designed and modeled. An attempt is made to reduce the amplitude of the reflected voltage in the feed transmission line. For one IRA, each feed arm is terminated with two resistors. For the other IRA, each feed arm is tapered and terminated with one resistor.

4.3.1 Model I: Two-Resistor Feed Arm Termination

For the feed structure of the offset IRA, a pair of curved plates [27] is considered. The curved plates cause significant aperture blockage when they are used in the center-fed geometry; however, they cause almost no blockage for the offset geometry.

Consider the geometry shown in Figure 4.29. The reflector is a portion of a paraboloid, which is defined as

\[ x^2 + y^2 = 4F(z + F), \]  \hspace{1cm} (4.3)

where \( F \) is the focal length of the paraboloid, and its focus is placed at the coordinate origin. The edge of the reflector is determined by the intersection with a cone, whose
Figure 4.29: Geometry of the offset IRA. (a) Projection onto $y$-$z$ plane. (b) Projection onto $x$-$z$ plane. (d) Projection onto $x$-$y$ plane. (c) Magnification of the region around the feed arm termination.
surface is defined as
\[
\sqrt{(x \cos \beta + z \sin \beta)^2 + y^2} = (x \sin \beta - z \cos \beta) \tan \chi,
\] (4.4)
where \( \chi = 29.7^\circ \) is the interior half angle of the cone. The apex of the cone is placed at the coordinate origin, and its rotational axis lies in the \( x-z \) plane making an angle \( \beta = 82.9^\circ \) with the \( z \)-axis. The intersection of the cone and the paraboloid is an ellipse and its projection on the \( x-y \) plane is a circle of diameter \( D = 2F \) [57].

The curved plates are placed on the surface of the cone such that they form a TEM feed structure with a characteristic impedance of 400\( \Omega \). Each plate is connected through two chip resistors \((R_1, R_2)\) to the paraboloidal reflector. These resistors are used as low frequency matching circuits. The resistance of each resistor can be adjusted, while the parallel resistance is kept at 200\( \Omega \), in an attempt to reduce the reflected voltage in the feeding transmission line.

A set of triangular meshes is generated for the antenna. To improve the efficiency of the numerical model, different meshes are used according to the frequency, and PEC symmetry is utilized noting the reflection symmetry across the \( x-z \) plane. The meshes contain 5590 triangle elements for the low frequency model and 11091 triangle elements for the high frequency model. The meshes are used to calculate the response of the antenna at 150 equally spaced points within the normalized frequency range of \( D/\lambda = 0.102 \) to \( D/\lambda = 15.3 \). The lower 75 and the upper 75 frequencies are calculated using the low frequency model and the high frequency model, respectively. The total run time is approximately 86.1 hours on the Beowulf cluster using 32 AMD Athlon\textsuperscript{TM} XP 2200+ processors.

In Figure 4.30, the reflected voltage in the feed transmission line is graphed as a function of normalized time \( t/\tau_a \). For the graphs, three pairs of termination resistors are used; \((400\Omega, 400\Omega)\), \((600\Omega, 300\Omega)\), and \((1000\Omega, 250\Omega)\), whose parallel resistances are all 200\( \Omega \). The input pulses for the graphs are a step-like pulse with \( t_{10-90\%} = 0.1\tau_a \) and a Gaussian pulse with \( t_{FWHM} = 0.1\tau_a \).
Figure 4.30: Comparisons of the reflected voltages. Input pulses are: (a) step-like $t_{10-90\%} = 0.1\tau_a$ and (b) Gaussian $t_{FWHM} = 0.1\tau_a$. 

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In the figures, small reflections are seen around \( t/\tau_a = 0 \). These reflections are due to the approximation of the apex geometry (Figure 4.33 (a)). The first possible disturbance after this is the signal from the edge of the reflector near \( t/\tau_a = 1.25 \). However, in Figure 4.30 (a), the waveform is not exactly zero over the time interval \( 0 < t/\tau_a < 1.25 \). This is due to a small error in the characteristic impedance of the TEM feed structure predicted by the model (\( \sim 1\% \)).

In the figure, \( T_{E1} \) and \( T_{E2} \) mark the first possible signals from the closest and farthest points on the reflector from the apex, and \( T_{R1} \) and \( T_{R2} \) mark the first possible signals from the resistors. Note that, in Figure 4.30 (b), the waveform near \( T_{R1} \) is negative and the waveform near \( T_{R2} \) is positive. This implies that the signals from around \( R1 \) and around \( R2 \) are reflections from lower impedance and higher impedance than the characteristic impedance of the TEM feed structure, respectively.

Varying the resistances of \( R1 \) and \( R2 \) changes the waveform slightly. As the resistance of \( R2 \) decreases, the amplitude of the signal near \( T_{R2} \) decreases. However, the waveform near \( T_{R1} \) is hardly changed. In addition, the amplitude of the signal over the interval \( T_{E1} - T_{R1} \) can not be changed by varying the resistances.

One way to reduce the amplitude of the signal over the interval \( T_{E1} - T_{R1} \) may be tapering the TEM feed arms appropriately. This will result in gradually increasing impedance along the tapered section and cause positive reflection. This positive reflection may compensate for the negative signal over the interval \( T_{E1} - T_{R1} \), as has done for centered geometries [58].

### 4.3.2 Model II: Tapered Feed Arm Termination

Consider the feed structure shown in Figure 4.31. The feed structure is described by an angle \( \alpha(r) \), which is measured in the projected geometry as shown in Figure 4.31 (a). The region inside a sphere of radius \( L \) centered at the apex is the undisturbed spherical TEM feed structure. Using Eq. (2.3) with Eq. (2.9), the impedance of the
Figure 4.31: Schematic of arm taper. (a) Projection onto the plane that is perpendicular to the cone axis. (b) Side view.
TEM feed structure for the curved plates is calculated from the angle $\alpha_0$ as [27]

$$Z_c = \eta_0 \frac{K(m)}{K(1 - m)}, \quad m = \left[1 - \frac{\sin(\alpha_0/2)}{\cos(\alpha_0/2)}\right]^4. \quad (4.5)$$

By assuming that only a TEM mode exists along the entire TEM feed arms, the characteristic impedance $Z_c(r)$ of a section that is between radii $r$ and $r + \Delta r$ can be obtained by replacing $\alpha_0$ in Eq. (4.5) with $\alpha(r)$. The reflected waveform can be approximately obtained from the characteristic impedance profile $Z_c(r)$.

Consider a linear taper of the angle $\alpha$:

$$\alpha(r) = \begin{cases} 
\alpha_0, & 0 < r \leq L \\
\alpha_0 + \frac{r - L}{L_A - L} (\alpha_{\text{min}} - \alpha_0), & L < r < L_A
\end{cases} \quad (4.6)$$

where $L_A$ is the total length of an arm. Here, $L$ is equal to the distance from the apex to the closest point on the reflector, and $\alpha_{\text{min}} = W_R/L_A$. For the IRAs in this section, the chip resistor width is $W_R = 0.8\text{mm}$ [59], and $D = 30.6\text{cm}$ is chosen for the aperture diameter.

Figure 4.32 shows the inverted reflected voltage (solid line) calculated from the impedance profile for a step-like input with $t_{10\%-90\%} = 0.1\tau_a$. For comparison, the dotted line is copied from the line for $(1000\Omega, 250\Omega)$ in Figure 4.30 (a). The lines are roughly on top of each other over the interval from $T_{E1}$ to $T_{R1}$, which implies the reflected signal from the taper is large enough to cancel the signal over $T_{E1} - T_{R1}$.

The high frequency model for the IRA with the tapered feed structure described by Eq. (4.6) is shown in Figure 4.33. Note that the PEC symmetry is utilized and only a half of the geometry is meshed. The meshes contain 5295 triangle elements for low frequency model and 10516 triangle elements for high frequency model. The total run time is approximately 76.1 hours on the Beowulf cluster using 32 AMD Athlon™ XP 2200+ processors.

The reflected voltage on the feed transmission line is shown in Figure 4.30. Note that the amplitude of the waveform is significantly reduced, and the polarity is
Figure 4.32: Reflected voltage from the tapered feed arms, which is calculated by the simple model using the impedance profile (solid line) and the reflected voltage from the non-tapered feed arms, which is calculated by the EIGER model (dotted line). Step-like pulse with $t_{10-90\%} = 0.1\tau_a$ is assumed to be incident through a 400Ω transmission line.

changed. The reflection from the taper slightly over compensates the signal from the reflector. The cancellation occurs in two mechanisms. The first mechanism is the reflection from the taper due to the impedance change. The effect appears over the interval from $T_{E1}$ to $T_{R1}$. The second mechanism is related to the reduction in the interaction of feed arms and the paraboloidal reflector. This effect appears near the interval from $T_{R1}$ to $T_{R2}$.

Compared with the IRAs with two-resistor terminations, the IRA with tapered feed arm termination shows better performance in terms of the amplitude of the reflected voltage. In the next section, the IRA with tapered feed arms is further analyzed to understand the radiation characteristics of the offset IRA.
Figure 4.33: Mesh for the offset IRA. (a) Magnification of the region around the feed apex. (b) Projection on to $x$-$z$ plane. (c) Magnification of the region around the feed arm termination. (d) Projection on to $x$-$y$ plane.
Table 4.3: Parameters defining geometries of the IRAs.

<table>
<thead>
<tr>
<th>Feed Structure</th>
<th>Parameter</th>
<th>F/D</th>
<th>L/D</th>
<th>Z₀</th>
<th>β</th>
</tr>
</thead>
<tbody>
<tr>
<td>Offset IRA</td>
<td></td>
<td>0.5</td>
<td>0.5</td>
<td>400Ω</td>
<td>82.9°</td>
</tr>
<tr>
<td>Center-fed IRA</td>
<td></td>
<td>0.5</td>
<td>0.5</td>
<td>400Ω</td>
<td>0°</td>
</tr>
</tbody>
</table>

4.3.3 Analysis

The properties of the offset IRA may be shown more clearly by comparison with a center-fed IRA. In this section, the characteristics of the offset IRA are compared with those of the center-fed IRA that has been analyzed numerically in Sec. 4.1. The number of arm pairs was two in Sec. 4.1. Here, the number of the arm pairs are reduced to one to make both IRAs have the same number of arms. Also, the IRAs are fed by a 400Ω-transmission line. For convenience, the geometries of the IRAs are summarized in Table 4.3.

For convenience of analysis, the coordinate origin is redefined. In Figure 4.34, the coordinate origins for the antennas are shown. For each antenna, a plane that is normal to the z-axis and passes through the point on the antenna with the largest z-coordinate is taken. The coordinate origin is located at the center of the disc formed by the projection of the reflector onto the plane. The distance from the new coordinate origin to an observer is designated as $R = |\vec{R}|$.

Note that $r = |\vec{r}|$ is the distance from the apex of the TEM feed structure to the observer. For the center-fed IRA, $r$ and $R$ are the same both in the far zone and in the near zone. Because the effect of the input signal fed into the antenna at $t = 0$ is observed by the observer at the retarded time $t_r = t - r/c = 0$, the retarded time is used to graph near field quantities as well as far field quantities.

First, the reflected voltages in the transmission line are compared in Figure 4.35 for a step-like pulse and a Gaussian pulse with pulse parameters $t_{10-90\%} = 0.1\tau_a$ and $t_{FWHM} = 0.1\tau_a$, respectively. In the figure, the amplitude of the reflected voltage for the offset IRA is lower than that for the center-fed IRA. Note that the TEM feed
Figure 4.34: Measure of distance in the near field. $r$ is the distance from the apex of the TEM feed structure to the observer. $R$ is the distance from the center of a disc to the observer. The disc is the a projection of the reflector to the plane that is normal to the $z$-axis and passes through the point on the antenna with the largest $z$-coordinate.
Figure 4.35: Comparisons of the reflected voltages for the offset and center-fed IRAs. Input pulses are (a) a step-like with $t_{10-90\%} = 0.1\tau_a$ and (b) a Gaussian with $t_{FWHM} = 0.1\tau_a$. 
arms in both IRAs are linearly tapered in an attempt to reduce the amplitudes of the reflected voltages by allowing the first signals from the TEM feed arm terminations to cancel the first signals from the reflectors. The cancellation is seen to be more effective for the offset IRA than for the center-fed IRA.

In Figure 4.36, the far-field waveforms on boresight for a step-like pulse with $t_{10-90\%} = 0.1\tau_a$ are compared. In the figure, the waveforms are normalized by $V_0/R$ and graphed as functions of time $t_r/\tau_a$. The prepulse and the impulse are very similar for both antennas. The prepulse height of the center-fed IRA is slightly higher than that of the offset IRA due to the difference in the TEM feed structure and the observation angle with respect to the axis of the TEM feed structure.

The postpulse waveforms are very different. The postpulse waveform of the center-fed IRA is bigger and more complicated than that of the offset IRA. Unlike the center-fed IRA, in the offset IRA, there is negligible blockage in the aperture by metal arms. Thus, the multiple reflections between the feed structure and the reflector are small, making the postpulse small and simple.
Figure 4.37 shows the radiated fields of the offset IRA and the center-fed IRA at different observation angles as functions of time. A step-like with \( t_{10-90\%} = 0.1\tau_a \) is used as an input pulse. Here, E- and H-planes are defined as the \( y-z \) and \( x-z \) planes, respectively. The angles are the ones made by \( z \)-axis and the position vector to an observer. The sign of an angle are the same as the sign of the observer’s \( x \) or \( y \) coordinates.

In Figure 4.37 (b), (d) the waveforms are all symmetric about 0° because the geometry of the center-fed IRA has two symmetries across the E- and H-planes. The offset IRA, however, has only one symmetry, across the H-plane. Thus, the waveforms in the E-plane are symmetric (Figure 4.37 (c)), but they are asymmetric in the H-plane (Figure 4.37 (a)).

Note that the radiated fields are seen to be minimal at 90° for the offset IRA and at 180° for the center-fed IRA. The reason for this is that the observer is located in the shadows of the reflectors at 82.9° for the offset IRA and at 180° for the center-fed IRA. This phenomenon may be useful in certain applications, such as the bistatic GPR. In a bistatic GPR system, the coupling between the antennas should be reduced significantly by placing two offset IRAs in the shadow of each other.

Figure 4.38 shows the prepulse and impulse heights as functions of observation angle for a range of step-like pulses. The maximum of the normalized far field \( (RE_r(t)/V_0) \) is taken for the impulse height and is plotted above zero, and the minimum is taken for the prepulse height and is plotted below zero. In the figure, the prepulse and impulse heights are larger for those with smaller \( t_{10-90\%} \). The impulse height of the offset IRA is slightly higher at small observation angles and lower at large observation angles than that of the center-fed IRA for each \( t_{10-90\%} \). This indicates that the directivity of the offset IRA is higher than that of the center-fed IRA for pulse radiation.

In Figure 4.39, near field waveforms are graphed as a function of retarded time
Figure 4.37: Radiated fields of offset and center-fed IRAs (a), (b) in the H-plane and (c), (d) in the E-plane. A step-like with $t_{10-90\%} = 0.1\tau_a$ is used.
Figure 4.38: Prepulse and impulse heights as functions of observation angle for a range of step-like pulses. The lines above zero are those of impulse heights and the lines below zero are those of prepulse heights. The lines that are farther from zero are those with shorter rise time. The graphs show the prepulse and impulse heights for the offset IRA observed (a) in the H-plane and (c) in the E-plane; and those for the center-fed IRA observed (b) in the H-plane and (d) in the E-plane.
for a step-like pulse with $t_{10-90\%} = 0.1\tau_a$. In the figure, the normalized electric field $(RE_y(R, t)/V_0)$ at a number of points along the $z$-axis is plotted. The development of the impulse is seen as the observer moves out. In Figure 4.39 (a), the prepulse height of the offset IRA varies because the observation angle seen by the apex of the TEM feed structure varies and the field is not normalized by the distance between the apex and the observer. In Figure 4.39 (b), the prepulse height of the center-fed IRA stays the same because of its $1/r$ dependence.

The length of the prepulse varies more for the offset IRA than for the center-fed IRA. For the offset IRA, the length of the prepulse is shorter at a point closer to the antenna because the difference between the path length of the signal from the apex and the path length of the signal from the reflector is smaller at a point closer to the antenna. For the center-fed IRA, length of the prepulse is slightly larger at a point closer to the antenna, and therefore the impulse is centered at slightly later than $t_r/\tau_a = 1.0$. This is related to the finite size of the aperture as shown in Sec. 4.2.

Figure 4.40 shows the spot sizes in the E-plane and in the H-plane. An imaginary plane orthogonal to both the E- and H-planes is placed at a coordinate $z$. Then, the location of the half maximum of the impulse amplitudes are plotted for each $z$. Step-like pulses with pulse parameters $t_{10-90\%}/\tau_a = 0.1, 0.15, 0.2$ are used. The solid lines are those of the offset IRA and the dotted lines are those for the center-fed IRA. The lines that are inner are those for the pulses with smaller $t_{10-90\%}$. In Figure 4.40 (a), the spots for the offset IRA are seen to be asymmetrical due to the asymmetry in the geometry. Also, the spots of the offset IRA are wider than those of the center-fed IRA both in the H-plane and in the E-plane.

In Figure 4.41, radiated power $P_{rad}$, dissipated power $P_{diss}$, and reflected power $P_{refl}$ are normalized by input power $P_{in}$ and graphed as functions of normalized frequency $D/\lambda$. Note the frequency where the radiated power and the dissipated power are equal. This frequency is lower for the center-fed IRA than for the offset
Figure 4.39: Near field waveforms of (a) the offset IRA and (b) the center-fed IRA. The distance $r$ along the $z$-axis is measured from the apex of the feed structure. A step-like with $t_{10-90\%} = 0.1\tau_a$ is used.
Figure 4.40: Spot sizes (a) in the H-plane and (b) in the E-plane. The input pulses are step-like with $t_{10-90%}/\tau_a = 0.1, 0.15, 0.2$ from inside.
Figure 4.41: Comparison of power budgets. Radiated power $P_{\text{rad}}$, dissipated power $P_{\text{diss}}$, and reflected power $P_{\text{refl}}$ are normalized by input power $P_{\text{in}}$ and graphed as functions of normalized frequency $D/\lambda$ for the two IRAs.
IRA. Since significant portion of the low frequency content radiates through the direct radiation off the TEM feed arms, the TEM feed arms of the offset IRA are believed to radiate less than the center-fed IRA.

At most of the frequencies, the center-fed IRA radiates more power and dissipates less power than the offset IRA. However, Figure 4.38 shows that the offset IRA has larger impulse amplitude than the center-fed IRA for each input pulse, implying that the directivity of the impulse from the offset IRA has to be higher even though the impulse beam widths of the offset IRA are seen to be wider in Figure 4.40.

4.3.4 Conclusion

Two offset IRAs were numerically modeled and analyzed. The first one had the TEM feed structure whose feed arms were terminated with two resistors at each connection to the reflector. The termination resistors could be adjusted to reduce the late-time amplitude of the reflected voltage in the feeding transmission line; this was not very effective.

The feed arms of the second IRA were tapered in an attempt to reduce the amplitude of the reflected voltage in the feeding transmission line. A simple model for the reflected voltage from the TEM feed structure was constructed. The simple model predicted that a linear taper would cancel the signal from the reflector and reduce the amplitude of the reflected voltage. The EIGER model proved that the taper scheme was effective in reducing the amplitude of the reflected voltage. This taper profile can be further refined to reduce the amplitude of the reflected voltage.

The offset IRA with the tapered feed arms was further analyzed and compared with the center-fed IRA, which was analyzed in Sec. 4.1. In the far field, the radiated waveforms and the prepulse and impulse heights were compared. In the near field, waveforms and spot sizes were compared. In the frequency domain, the power budgets were compared as functions of normalized frequency.
The postpulse of the offset IRA was lower in amplitude and simpler in shape than that of the center-fed IRA resulting from the reduction in the multiple reflections between the TEM feed arms and the reflector. The analysis of the offset IRA showed the asymmetry in the radiated fields due to the asymmetry in the geometry. The offset IRA radiated less power than the center-fed IRA. However, the offset IRA reflected less power back into the feeding transmission line than the center-fed IRA.

4.4 TEM Feed Arm Termination

In this section, the effect of the shape of the TEM feed arm terminations on the performance of the IRA is investigated. Two factors are considered: the beginning point of the tapered termination (the choice of $L$ – see Figure 4.42) and the shape of the taper.

Note that the region inside a sphere of radius $L$ centered at the apex of TEM feed arms is a spherical TEM waveguide. Thus, the taper and the chip resistor form a termination of the TEM waveguide. The wave propagating outward in the spherical TEM waveguide does not experience discontinuity. The first possible discontinuity occurs either at the beginning of the termination or at the reflector, and these discontinuities cause reflections. In [58], Baum expected that the choice $L = F$ would allow the reflection from the taper to offset the reflection from the reflector. Also, In Sec. 4.3, it was shown that the choice $L = F$ lowers the reflected voltage in the feeding transmission line.

In the previous sections, only the choice of $L = F$ and linear taper were considered. In this section, other choices of $L$ and other shape of the taper will be considered. To simplify the analysis, we will choose a set of curves for the taper that can be described by one mathematical expression.
4.4.1 Length of Termination

To investigate the effects of the length of termination when the shape of the termination is a linear taper, consider the geometries shown in Figure 4.42. The angles associated with the feed arms are chosen such that the characteristic impedance is $Z_0 = 400\Omega$. Each feed arm is terminated by a $200\Omega$ resistor as a low frequency matching circuit. After a radial distance $L$ from the apex, the arms are linearly tapered to the resistors. Four IRAs with linear tapered TEM feed arms are numerically modeled. The lengths of the spherical TEM waveguides are $L = F/2, 7F/8, F, \text{ and } 9F/8$.

Triangular meshes are generated for the antennas. To improve the efficiency of the numerical model, different meshes are used depending on the frequency, and PEC symmetry is utilized noting the reflection symmetry across the $x$-$z$ plane. The high frequency meshes contain 7387, 7826, 7954, and 8102 triangle elements, and the low frequency meshes contain 4661, 4842, 4918, and 4993 triangle elements for the IRAs with $L = F/2, 7F/8, F, \text{ and } 9F/8$, respectively.

The meshes are used to calculate the response of the antenna at 120 equally spaced points within the normalized frequency range of $D/\lambda = 0.102$ to $D/\lambda = 12.3$. The lower 75 and the upper 45 frequencies are calculated using the low frequency model and the high frequency model, respectively.

The graphs in Figure 4.43 (a), (b) show the reflected voltages in the feeding transmission line as functions of time for a step-like pulse with $t_{10-90\%/\tau_a} = 0.12$ and a Gaussian pulse with $t_{FWHM}/\tau_a = 0.12$. The first possible signals from the reflector appear around $t/\tau_a = 1$. In the figure, the first pulse around $t/\tau_a = 0$ is the reflection from the apex approximation in the model. After the first pulse, the waveform is seen to have nonzero value before the first possible signals from the TEM feed arm terminations. This is due to a small error in the characteristic impedance of the TEM feed structure predicted by the model (1%). The first possible signals from the TEM feed arm terminations appear around $t/\tau_a = 0.5, 0.875, 1.0, \text{ and } 1.125$ for IRAs with
Figure 4.42: Geometries for TEM feed arms with linear taper. (a) Geometry of an IRA with TEM feed arms with $L = 7F/8$. (b) The arms used for the comparison.
Figure 4.43: Reflected voltages in the feeding transmission line as functions of time for (a) a step-like pulse with $t_{10-90\%}/\tau_a = 0.12$ and (b) a Gaussian pulse with $t_{FWHM}/\tau_a = 0.12$. Times of the first possible signals are marked with dotted lines.
\( L = F/2, 7F/8, F, \) and \( 9F/8, \) respectively.

In Figure 4.43 (a), for the IRAs with \( L = F/2 \) and \( 7F/8, \) the signals from the TEM feed arm termination appear first, which is positive, and, for the IRA with \( L = 9F/8, \) the signal from the reflector appears first, which is negative. Note that for the IRA with \( L = F, \) the reflected voltage begins with a negative signal. This implies that the reflected signal from the TEM feed arm termination is weaker than the signal from the reflector for the geometry shown in Figure 4.42. Note also that, in Figure 4.43 (a), the peak-to-peak amplitude is the lowest for the IRA with \( L = 7F/8. \)

The graphs in Figure 4.44 (a), (b) show the radiated field on boresight as functions of time for a step-like pulse with \( t_{10-90\%}/\tau_a = 0.12 \) and a Gaussian pulse with \( t_{FWHM}/\tau_a = 0.12. \) Notice that the prepulse is distorted near the impulse. The distortion begins earlier for the TEM feed arms with smaller value of \( L. \) Because the prepulse is the direct radiation off the TEM feed arms, the discontinuity in the TEM feed arm termination causes distortion in the direct radiation.

The postpulse waveforms also depend on \( L. \) In Figure 4.44 (a), the peak-to-peak amplitude of the postpulse for the IRA with \( L = F/2 \) is represented as \( A_T, \) which is the difference between the maximum and minimum of the signal amplitude after the impulse. In the figure, it is shown that \( A_T \) is the smallest for the IRA with \( L = F. \)

### 4.4.2 Shape of Termination

In the previous section, the reflected voltages in the feeding transmission line and the radiated fields on boresight for the IRAs with linear tapers were compared. In this section, nonlinear tapers are considered.

Consider the geometry shown in Figure 4.45. The figure shows a TEM feed arm, which is terminated with a curved taper after a spherical TEM waveguide of length \( L. \) The taper curve will be described by the width \( y, \) which is measured from the reference line, as a function of distance \( x \) from the end of the TEM waveguide. Two
Figure 4.44: Radiated fields on boresight as functions of time for (a) a step-like pulse with $t_{10-90\%}/\tau_a = 0.12$ and (b) a Gaussian pulse with $t_{\text{FWHM}}/\tau_a = 0.12$. 
The taper curve is constrained by

\[ y = y_{\text{max}} \quad \text{at} \quad x = 0, \quad (4.7) \]

\[ y = y_{\text{min}} \quad \text{at} \quad x = x_{\text{max}}, \]

where \( y_{\text{max}} \) is the width of the TEM feed arm at the end of the TEM waveguide measured from the reference line, and \( y_{\text{min}} \) is the half width of the chip resistor. The equation for the taper curve should also be controlled by the slope at the beginning
of the termination. These conditions can be met with the following equation:

\[ y = \frac{1}{p} \ln(ax + b) + mx, \tag{4.8} \]

where the parameters are defined for given \( p \) as

\[
\begin{align*}
    a &= \frac{1}{x_{\text{max}}} \left[ e^{p(y_{\text{min}}-mx_{\text{max}})} - e^{yp_{\text{max}}} \right], \\
    b &= e^{yp_{\text{max}}}, \\
    m &= \begin{cases} 
        m_0, & p > 0, \\
        0, & p \leq 0,
    \end{cases}
\end{align*}
\tag{4.9}
\]

where \( m_0 \) is the same as the slope of the conical TEM waveguide. Note that Eq. (4.8) with Eq. (4.9) satisfies the conditions in Eq. (4.7) irrespective of \( p \).

Asymptotically, Eq. (4.8) becomes

\[
\begin{align*}
    y &\to y_{\text{max}} + mx + \frac{1}{p} \ln \left(1 - \frac{x}{x_{\text{max}}} \right), \quad \text{as } p \to \infty, \tag{4.10} \\
    y &\to \frac{y_{\text{min}} - y_{\text{max}}}{x_{\text{max}}} x + y_{\text{max}}, \quad \text{as } p \to 0, \tag{4.11} \\
    y &\to y_{\text{min}} + \frac{1}{p} \ln \frac{x}{x_{\text{max}}}, \quad \text{as } p \to -\infty. \tag{4.12}
\end{align*}
\]

Eq. (4.8) are plotted for these limiting cases in Figure 4.45 (b) along with the lines for typical values of \( p \). The slope of the curve at the beginning of the termination is described as

\[
\tan \theta = \left. \frac{dy}{dx} \right|_{x=0} = -\frac{1}{px_{\text{max}}} \left[ 1 - e^{-p(y_{\text{max}}-y_{\text{min}}+mx_{\text{max}})} \right] + m. \tag{4.13}
\]

For a specific slope, \( p \) can be found numerically.

Two curves are chosen for the numerical model: one with a negative \( p \) and one with a positive \( p \). They are described in terms of the angle between the x-axis and the tangent line to the curve at the beginning of the termination. The angles are chosen for a smooth termination and a sharp termination as

\[
\begin{align*}
    \theta_{\text{smooth}} &= \frac{\tan^{-1}(m_0) + \theta_{\text{ref}}}{2}, \tag{4.14} \\
    \theta_{\text{sharp}} &= \frac{-3\pi/2 + \theta_{\text{ref}}}{4}. \tag{4.15}
\end{align*}
\]
For each shape of termination, the angles are calculated twice because each termination requires two curves: above and below the reference line. The parameter $p$’s are found by solving Eq. (4.13) using the angles found in Eq. (4.15). The results are shown in Table 4.4, where $p$’s are calculated for the angles calculated in Eq. (4.15) for $L = 7F/8$, $L = F$, and $L = 9F/8$. The parameters $p_d$ and $p_u$ are those for the curve below the reference line and the curve above the reference line.

The resulting shapes of the TEM feed arms are shown in Figure 4.46. In the figure, two shapes are drawn for each $L$: the upper one is drawn with a negative $p$, and the lower one is drawn with a positive $p$. The angles associated with the TEM feed arms are chosen such that the characteristic impedance is $Z_0 = 400\Omega$. For the IRAs, each TEM feed arm is terminated by a $200\Omega$ resistor as a low frequency matching circuit. The response of the IRAs with these nonlinearly terminated TEM feed arms are numerically modeled and compared with the responses of the IRAs with the linearly tapered TEM feed arms.

Triangular meshes are generated for the antennas. To improve the efficiency of the numerical model, different meshes are used according to the frequency, and PEC symmetry is utilized noting the reflection symmetry. The number of triangle elements used for the IRAs are shown in Table 4.5.

The meshes are used to calculate the response of the antenna at 150 equally spaced points within the normalized frequency range of $D/\lambda = 0.102$ to $D/\lambda = 15.3$, where $\lambda$
Figure 4.46: Shapes of TEM feed arms with $L = 7F/8$, $F$, and $9F/8$. A shape with a negative $p$ and a shape with a positive $p$ are drawn for each $L$.

Table 4.5: Number of Triangle Elements

<table>
<thead>
<tr>
<th></th>
<th>$L = 7F/8$</th>
<th>$L = F$</th>
<th>$L = 9F/8$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$p &lt; 0$</td>
<td>$p &gt; 0$</td>
<td>$p &lt; 0$</td>
</tr>
<tr>
<td>high frequency</td>
<td>7674</td>
<td>7961</td>
<td>7853</td>
</tr>
<tr>
<td>low frequency</td>
<td>4788</td>
<td>4917</td>
<td>4871</td>
</tr>
</tbody>
</table>
is the free space wavelength. The lower 75 and the upper 75 frequencies are calculated using the low frequency model and the high frequency model, respectively.

Figures 4.47 - 4.49 show the reflected voltages in the feeding transmission lines of the IRAs for a step-like pulse with $t_{10-90%}/\tau_a = 0.12$ and a Gaussian pulse with $t_{FWHM}/\tau_a = 0.12$. In each figure, three waveforms are shown for three shapes of terminations, which are $p > 0$, $p = 0$, and $p < 0$.

In the figures, the first pulse around $t/\tau_a = 0$ is the reflection from the apex approximation in the model. After the first pulse, the waveform is seen to have nonzero value before the first possible signals from the TEM feed arm terminations, which begin at $t/\tau_a = 0.875$ in Figure 4.47, $t/\tau_a = 1.0$ in Figure 4.48, and $t/\tau_a = 1.125$ in Figure 4.49. This is due to a small error in the characteristic impedance of the TEM feed structure predicted by the model ($\sim 1\%$).

In Figure 4.47, the positive signals from the TEM feed arm terminations begin around $t/\tau_a = 0.875$, and the negative signals from the reflectors begin to offset the positive reflection around $t/\tau_a = 1.0$. Thus, in Figure 4.47 (a), the waveform begins with positive pulse. Because the termination with a sharp curve ($p < 0$) tends to reflect more current than the termination with a smooth curve ($p > 0$), the leading edge of the waveform is steep with $p < 0$. Note that the peak-to-peak amplitude is the smallest for $p = 0$.

In Figure 4.48, the positive signals from the TEM feed arm terminations and the negative signals from the reflectors begin at the same time around $t/\tau_a = 1.0$. Thus, in Figure 4.48 (a), the leading edge depends on the amount of the current reflection from the termination. The peak-to-peak amplitude is the smallest for $p < 0$.

In Figure 4.49, the positive signals from the TEM feed arm terminations begin around $t/\tau_a = 1.125$, and the negative signals from the reflectors begin to offset the positive reflection around $t/\tau_a = 1.0$. Thus, in Figure 4.49 (a), the waveform begins with negative pulse, and the peak-to-peak amplitude is the smallest for $p < 0$. 

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Figure 4.47: Reflected voltages in the feeding transmission line as functions of
time for (a) a step-like pulse with $t_{10-90\%}/\tau_a = 0.12$ and (b) a Gaussian pulse with
$t_{FWHM}/\tau_a = 0.12$. The antennas have three shapes of TEM feed arm terminations
with $L = 7F/8$. 

![Graph showing reflected voltages for step-like and Gaussian pulses.](image-url)
Figure 4.48: Reflected voltages in the feeding transmission line as functions of time for (a) a step-like pulse with $t_{10-90%}/\tau_a = 0.12$ and (b) a Gaussian pulse with $t_{FWHM}/\tau_a = 0.12$. The antennas have three shapes of TEM feed arm terminations with $L = F$. 
Figure 4.49: Reflected voltages in the feeding transmission line as functions of time for (a) a step-like pulse with $t_{10-90%}/\tau_a = 0.12$ and (b) a Gaussian pulse with $t_{FWHM}/\tau_a = 0.12$. The antennas have three shapes of TEM feed arm terminations with $L = 9F/8$. 
For a comparison, the reflected voltage waveforms with the smallest peak-to-peak amplitudes in Figures 4.47 – 4.49 are plotted in Figure 4.50. In Figure 4.50, the waveform with \( L = 9F/8 \) and \( p < 0 \) may be the simplest in shape, but the amplitude is the highest. The smallest amplitude is seen to be the one with \( L = F \) and \( p < 0 \).

Figures 4.51 – 4.53 show the radiated field on boresight of the IRAs for a step-like pulse with \( t_{10-90\%}/\tau_a = 0.12 \) and a Gaussian pulse with \( t_{FWHM}/\tau_a = 0.12 \). In each figure, the radiated field is plotted for each IRA, whose TEM feed arm terminations are described by parameter \( p \).

In Figure 4.51, the prepulse distortions begin around \( t_r/\tau_a = 0.657 \). Because the sharp termination \( (p < 0) \) causes more disturbance in the prepulse than the smooth termination \( (p > 0) \), the prepulse distortion is stronger with \( p < 0 \). The impulse amplitudes does not seem to vary much with \( p \)'s, but the postpulse amplitudes vary much. In the figure, the peak-to-peak amplitude of the postpulse is the smallest for \( p = 0 \).

In Figure 4.52, the prepulse distortions begin around \( t_r/\tau_a = 0.751 \), and in Figure 4.53, the prepulse distortions begin around \( t_r/\tau_a = 0.844 \). In Figure 4.53, the prepulse is hard to distinguish, because the prepulse distortions begin close to the impulse. The peak-to-peak amplitudes of the postpulses are the smallest for \( p = 0 \) in Figure 4.52 and \( p < 0 \) in Figure 4.53.

For a comparison, the radiated fields with the smallest peak-to-peak postpulse amplitudes in Figures 4.51 – 4.53 are plotted in Figure 4.54. In Figure 4.54, the prepulse distortion is the smallest for \( L = 9F/8 \) and \( p < 0 \). The impulse height is also the highest for \( L = 9F/8 \) and \( p < 0 \), although the difference is only about 2.5% and may be meaningless deviation for practical antennas. The smallest postpulse amplitude corresponds to \( L = F \) and \( p = 0 \).
Figure 4.50: Comparison of the reflected voltage waveforms. The waveforms with the smallest peak-to-peak amplitudes in Figures 4.47 – 4.49 are plotted.
Figure 4.51: Radiated fields on boresight as functions of time for (a) a step-like pulse with $t_{10-90\%}/\tau_a = 0.12$ and (b) a Gaussian pulse with $t_{FWHM}/\tau_a = 0.12$. The antennas have three shapes of TEM feed arm terminations with $L = 7F/8$. 
Figure 4.52: Radiated fields on boresight as functions of time for (a) a step-like pulse with $t_{10-90%}/\tau_a = 0.12$ and (b) a Gaussian pulse with $t_{FWHM}/\tau_a = 0.12$. The antennas have three shapes of TEM feed arm terminations with $L = F'$. 
Figure 4.53: Radiated fields on boresight as functions of time for (a) a step-like pulse with $t_{10-90\%/\tau_a} = 0.12$ and (b) a Gaussian pulse with $t_{FWHM}/\tau_a = 0.12$. The antennas have three shapes of TEM feed arm terminations with $L = 9F/8$. 

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure.png}
\caption{Radiated fields on boresight as functions of time for (a) a step-like pulse with $t_{10-90\%/\tau_a} = 0.12$ and (b) a Gaussian pulse with $t_{FWHM}/\tau_a = 0.12$. The antennas have three shapes of TEM feed arm terminations with $L = 9F/8$.}
\end{figure}
Figure 4.54: Comparison of Radiated fields on boresight. (a) The waveforms with the smallest peak-to-peak postpulse amplitudes in Figures 4.51 – 4.53 are plotted. (b) Magnification of the box in (a).
4.4.3 Conclusion

The IRAs with a number of terminations were numerically modeled and analyzed in the time domain. The terminations of the TEM feed arms were described by a set of curves that has one degree of freedom, which can be used to control the smoothness at the discontinuity.

The reflected voltages in the feeding transmission line and the radiated fields on boresight were plotted and compared. The best ones in terms of the reflected voltage amplitude and in terms of the postpulse amplitudes were selected for each $L$ and compared.

Based on the results shown in Figures 4.50 and 4.54, the TEM feed arms with $L = F$ with $p < 0$ showed the best performance in terms of the reflected voltage amplitude, and the TEM feed arms with $L = F$ with $p = 0$ showed the best performance in terms of the postpulse amplitude. The shape of the TEM feed arm termination has a strong effect on the reflected voltage; however, the shape has a small effect on the radiated waveform.

4.5 Summary

In this chapter, a number of IRAs were numerically modeled and analyzed. The numerical model was developed using the method of moments as implemented in the EIGER code suite. The results were presented mainly in the time domain.

In Sec. 4.1, two reflector-type impulse-radiating antennas with $F/D$ of 0.25 and 0.5 were numerically analyzed and compared. With a larger $F/D$, the prepulse was longer and lower as expected in the simple model. The radiation efficiency at low frequency was higher with a larger $F/D$ because the prepulse was longer. The postpulse amplitude was lower with a larger $F/D$.

In Sec. 4.2, three impulse-radiating antennas with ellipsoidal reflectors have been
numerically modeled and compared with an impulse-radiating antenna with a parabolic reflector. The ellipsoidal reflector was parameterized by the distance between the center of the reflector and its closest focus ($F$), the distance between the foci ($\Phi$), and the diameter ($D$). The impulse-radiating antennas had the same $F/D$, but different $\Phi/D$'s, e.g., 0.5, 1.0, and 1.5. The apex of the TEM feed structure was located at the focus closer to the reflector. The maximum impulse height and the minimum spot size depended on the pulse rise time as well as $\Phi$, and they did not occur at the optical foci. However, the spot sizes of the focused IRA ($\Phi$) were smaller at distances close to the antennas than the spot sizes of the unfocused IRA ($\Phi = \infty$).

As a way to reduce the aperture blockage in the IRA geometry, an offset IRA was considered in Sec. 4.3. Two offset IRAs were modeled. For the first offset IRA, each TEM feed arm was terminated with two resistors, and for the second offset IRA, each TEM feed arm was tapered to one resistor. The reflected voltages in the feeding transmission line were compared, and the comparison showed that the offset IRA with tapered feed arms performed better. The IRA with tapered feed arms was further analyzed and compared with a center-fed IRA. The postpulse of the offset IRA was lower in amplitude and simple in shape than that of the center-fed IRA because of the reduction in the multiple reflections between the TEM feed structure and the reflector. The radiation pattern of the offset IRA was asymmetric in the H-plane due to the asymmetry in the geometry. The offset IRA radiated less power than the center-fed IRA. However, the offset IRA reflected less power back into the feeding transmission line than the center-fed IRA.

In Sec. 4.4, IRAs with a number of terminations were investigated. The terminations of the TEM feed arms were described by a set of curves that has one degree of freedom, which can be used to control the smoothness at the discontinuity. The reflected voltages in the feeding transmission line and the radiated fields on boresight were plotted and compared. The best ones in terms of the reflected voltage amplitude
and in terms of the postpulse amplitudes were selected for each $L$ and compared. The TEM feed arms with $L = F$ with $p < 0$ showed the best performance in terms of the reflected voltage amplitude, and the TEM feed arms with $L = F$ with $p = 0$ showed the best performance in terms of the postpulse amplitude. The shape of the TEM feed arm termination had a strong effect on the reflected voltage; however, the shape had a small effect on the tail waveform.
CHAPTER V

RESISTIVE VEE DIPOLES

5.1 Introduction

A resistive vee dipole (RVD) [19–21] is a dipole bent in an angle with a resistive profile on the dipole arms (Figure 5.1). The resistive profile considered in this chapter for the RVD is the Wu-King profile [12], for which the current on a dipole arm travels away from the drive point without internal reflections. The Wu-King profile is described by an internal impedance per unit length:

\[ R_i(r) = \frac{R_0}{1 - |r/h|}, \quad R_0 = \frac{\eta_0 \Psi}{2\pi h}. \] (5.1)

where \( R_0 \) is the impedance per unit length at the drive point, \( r \) is the distance along the arms from the drive point, \( \eta_0 \) is the wave impedance of free space, and \( h \) is the length of a dipole arm. The parameter \( \Psi \) was originally defined as

\[ \Psi = 2 \int_0^h \left( 1 - \frac{r'}{h} \right) e^{-jk(r'h + a^2)} e^{-jk\sqrt{r^2 + a^2}} dr', \] (5.2)

where \( a \) is the radius of the dipole arm, and \( k \) is the wave number. It is usually approximated for practical antennas by letting \( k = 0 \):

\[ \Psi_0 = 2 \ln \left[ \frac{h}{a} + \sqrt{\left( \frac{h}{a} \right)^2 + 1} \right] + 2 \left[ \frac{a}{h} - \sqrt{\left( \frac{a}{h} \right)^2 + 1} \right]. \] (5.3)

However, for pulse radiation applications, where an antenna must operate over a wide bandwidth, the parameter \( \Psi_0 \) can be determined by trade-off between the radiation efficiency and the amplitude of the tail following the main pulse in the radiated field.

A RVD with the Wu-King profile inherits characteristics from its geometry and loading profile. It is geometrically simple and directive [60], and it operates over a
**Figure 5.1:** Geometry of a RVD. The dipole of total length $2h$ is bent in angle $2\alpha$. The internal resistance per unit length varies as a function of distance from drive point.
broad bandwidth with low radar cross section, which prevents ringing that is caused by the multiple reflections between an antenna and its target in close proximity. It is also known to have the ability to radiate a short pulse whose shape is simply related to the input signal.

In the next section, the time-domain operation of RVDs with a Wu-King profile is described.

### 5.2 Resistive Vee Dipole with Wu-King Profile

The radiated electric field due to a line current distribution along a curve $C$ is described as [61]

$$E^r(\vec{r}, t) = -\frac{\mu_0}{4\pi r} \int_C \left[ \frac{\partial I(s', t')}{\partial t'} \right]_{t'=t_r} [\hat{s}' - \hat{r}(\hat{r} \cdot \hat{s}')] ds', \quad (5.4)$$

where $I(s', t)$ the current at point $s'$, $\hat{s}'$ is the unit vector tangential to curve $C$ at that point, and $\hat{r}$ is the unit vector from the coordinate origin to the observer. The retarded time is

$$t_r = t - (r - \hat{r} \cdot \hat{r}')/c, \quad (5.5)$$

where $\hat{r}'$ is the unit vector from the coordinate origin to point $s'$.

Consider the geometry of a RVD shown in Figure 5.1. It can be approximated as two loaded wires of length $h$ placed in vee shape with interior half angle $\alpha$. With a proper parameter $\Psi_0$, the current distribution along the left arm may be approximated as

$$I_+(r, t) = I_0(t - \hat{r}'/c)(1 - r'/h)[u(r') - u(r' - h)], \quad (5.6)$$

where $I_0(t)$ is the source current, and $u(r')$ is the Heaviside unit step function. The vector term in the bracket of Eq. (5.4) becomes

$$\hat{s}' - \hat{r}(\hat{r} \cdot \hat{s}') = \hat{r}' - \hat{r} \cos(\theta - \alpha); \quad (5.7)$$
so the radiated electric field of the element becomes

$$E_r^e(\vec{r}, t) = -\frac{\mu_0}{4\pi r} \int_{r'=0}^{h} \left(1 - \frac{r'}{h}\right) \left[ \frac{\partial I_0(t'-r'/c)}{\partial t'} \right]_{t'=t_r} [\vec{r'} - \hat{r} \cos(\theta - \alpha)] dr'. \quad (5.8)$$

The current expression in the bracket of the above equation can also be expressed as

$$\left[ \frac{\partial I_0(t'-r'/c)}{\partial t'} \right]_{t'=t_r} = -\frac{c}{1 - \cos(\theta - \alpha)} \frac{dI_0}{dt'} \left(t - \frac{r}{c} - \frac{r'}{c} \left[1 - \cos(\theta - \alpha)\right]\right). \quad (5.9)$$

Thus, after a few simple steps, the radiated field due to the current distribution becomes

$$E_r^e(\vec{r}, t) = \hat{\theta} \frac{\mu_0 c}{4\pi r} \sin(\theta - \alpha) \left\{ I_0 \left(t - \frac{r}{c}\right) - \frac{1}{h} \int_{r'=0}^{h} I_0 \left(t - \frac{r}{c} - \frac{r'}{c} \left[1 - \cos(\theta - \alpha)\right]\right) dr' \right\}. \quad (5.10)$$

Similarly, the radiated field due to the current distribution along the right arm becomes

$$E_r^e(\vec{r}, t) = -\hat{\theta} \frac{\mu_0 c}{4\pi r} \sin(\theta + \alpha) \left\{ I_0 \left(t - \frac{r}{c}\right) - \frac{1}{h} \int_{r'=0}^{h} I_0 \left(t - \frac{r}{c} - \frac{r'}{c} \left[1 - \cos(\theta + \alpha)\right]\right) dr' \right\}. \quad (5.11)$$

By adding the above two expressions, the expression for the radiated field of the RVD is obtained:

$$E_r^e(\vec{r}, t) = \hat{\theta} \frac{\mu_0 c}{4\pi r} \left\{ \frac{\sin(\theta - \alpha)}{1 - \cos(\theta - \alpha)} - \frac{\sin(\theta + \alpha)}{1 - \cos(\theta + \alpha)} \right\} I_0 \left(t - \frac{r}{c}\right) - \frac{\sin(\theta - \alpha)}{h [1 - \cos(\theta - \alpha)]} \int_{r'=0}^{h} I_0 \left(t - \frac{r}{c} - \frac{r'}{c} \left[1 - \cos(\theta - \alpha)\right]\right) dr' + \frac{\sin(\theta + \alpha)}{h [1 - \cos(\theta + \alpha)]} \int_{r'=0}^{h} I_0 \left(t - \frac{r}{c} - \frac{r'}{c} \left[1 - \cos(\theta + \alpha)\right]\right) dr'. \quad (5.12)$$

Eq. (5.12) can be calculated easily with a computer. Figure 5.2 (a) shows the radiated waveforms calculated using Eq. (5.12) for an RVD with $2\alpha = 60^\circ$. The graph shows the waveforms at different observation angles when $I_0(t)$ is a Gaussian pulse with $t_{FWHM} = 0.15\tau_a$. The waveforms calculated using Eq. (5.12) are compared with those obtained from a numerical model shown in Figure 5.2 (b). The numerical
Figure 5.2: Radiation pattern of the RVD for a Gaussian pulse with $t_{FWHM} = 0.15 \tau_a$. (a) Calculated using the analytical model. (b) Calculated using the numerical model. The interior angle of the RVD is $60^\circ$. For the numerical model, $R_0 = 1526 \Omega/m$ and the input pulse is incident in a 100Ω transmission line.
model was developed using EIGER. The RVD with $R_0 = 1526\Omega/m$ was fed by a 100$\Omega$ transmission line in the numerical model. The radiation patterns shown in the graphs are quite similar. The radiation patterns are directive in the direction of $\theta = 0$. The electric fields at $\theta = 0$ resemble the differentiated Gaussian pulse, which is the derivative of $I_0(t)$.

Figure 5.3 shows the peak-to-peak amplitudes of the radiated field at $\theta = 0$ calculated from the analytical model of the RVD for five Gaussian pulses as functions of $\alpha$. The interior half angle that results in the highest peak-to-peak amplitude for each Gaussian pulse is marked with a dot. For example, when the RVD is excited with a Gaussian pulse with $t_{FWHM} = 0.15\tau_a$, the interior half angle of about 31° will results in the highest peak-to-peak amplitude.

Smith showed that this approach was valid for an insulated linear antenna that is surrounded by material whose relative permittivity is greater than that of the insulation. The RVD can also be insulated. Let us assume that the current of the resistively loaded vee travels at a constant speed $v_e$, which is defined as

$$v_e = \frac{c}{\sqrt{\epsilon_{re}}} \quad (5.13)$$

where $\epsilon_{re}$ is the effective relative permittivity of the insulating material. Assume the current distribution on a basic traveling-wave element is

$$I_+(r, t) = I_0(t - r'/v_e)(1 - r'/h)[u(r') - u(r' - h)], \quad (5.14)$$

which is similar to Eq. (5.6) in the form. The radiated electric field of the element becomes

$$\bar{E}_r(r, t) = -\frac{\mu_0}{4\pi r} \int_{r' = 0}^{h} \left(1 - \frac{r'}{h}\right) \left[\frac{\partial I_0(t' - r'/v_e)}{\partial t'}\right]_{t'=t_r} [\hat{r}' - \hat{r} \cos(\theta - \alpha)] dr'. \quad (5.15)$$

The current expression in the bracket of the above equation can also be expressed as

$$\left[\frac{\partial I_0(t' - r'/v_e)}{\partial t'}\right]_{t'=t_r} = \frac{-c}{\sqrt{\epsilon_{re}} - \cos(\theta - \alpha)} \frac{dI_0}{dr'} \left(t - \frac{r}{c} - \frac{r'}{c} (\sqrt{\epsilon_{re}} - \cos(\theta - \alpha))\right). \quad (5.16)$$

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Figure 5.3: Peak-to-Peak amplitudes of the radiated field calculated from the analytical model of the RVD for a range of Gaussian pulses. The interior half angle that results in the highest peak-to-peak amplitude for each Gaussian pulse is marked with a dot.
After a few simple steps, the radiated field due to the current distribution becomes

\[
\vec{E}_r^\pm(\vec{r}, t) = \hat{\theta} \frac{\mu_0 c}{4\pi r} \frac{\sin(\theta - \alpha)}{\sqrt{\epsilon_{re} - \cos(\theta - \alpha)}} \left\{ I_0 \left( t - \frac{r}{c} \right) - \frac{1}{h} \int_{r' = 0}^{h} I_0 \left( t - \frac{r}{c} - \frac{r'}{c} \left[ \sqrt{\epsilon_{re} - \cos(\theta - \alpha)} \right] \right) dr' \right\}.
\]

(5.17)

Similarly, the radiated field due to the current distribution along the other basic traveling-wave element becomes

\[
\vec{E}_r^\mp(\vec{r}, t) = -\hat{\theta} \frac{\mu_0 c}{4\pi r} \frac{\sin(\theta + \alpha)}{\sqrt{\epsilon_{re} - \cos(\theta + \alpha)}} \left\{ I_0 \left( t - \frac{r}{c} \right) - \frac{1}{h} \int_{r' = 0}^{h} I_0 \left( t - \frac{r}{c} - \frac{r'}{c} \left[ \sqrt{\epsilon_{re} - \cos(\theta + \alpha)} \right] \right) dr' \right\}.
\]

(5.18)

By adding the above two expressions, one obtains the radiated field in the plane of the RVD:

\[
\vec{E}_r(\vec{r}, t) = \hat{\theta} \frac{\mu_0 c}{4\pi r} \left\{ \left[ \frac{\sin(\theta - \alpha)}{\sqrt{\epsilon_{re} - \cos(\theta - \alpha)}} - \frac{\sin(\theta + \alpha)}{\sqrt{\epsilon_{re} - \cos(\theta + \alpha)}} \right] I_0 \left( t - \frac{r}{c} \right) - \frac{\sin(\theta - \alpha)}{h} \int_{r' = 0}^{h} I_0 \left( t - \frac{r}{c} - \frac{r'}{c} \left[ \sqrt{\epsilon_{re} - \cos(\theta - \alpha)} \right] \right) dr' \right. \\
+ \left. \frac{\sin(\theta + \alpha)}{h} \int_{r' = 0}^{h} I_0 \left( t - \frac{r}{c} - \frac{r'}{c} \left[ \sqrt{\epsilon_{re} - \cos(\theta + \alpha)} \right] \right) dr' \right\}.
\]

(5.19)

Figure 5.4 (a) shows the radiated waveforms of the RVD with \(\alpha = 30^\circ\) at different observation angles for a Gaussian pulse with \(t_{FWHM} = 0.15\tau_a\). The effective relative permittivity of the insulation is \(\epsilon_{re} = 1.5\). Figure 5.4 (b) shows the radiated waveforms obtained from the EIGER model. The two radiation patterns are quite similar. The radiation pattern shown in Figure 5.4 are seen to differ from that shown in Figure 5.2. The shape of the waveforms are much different with small \(|\theta|\). The amplitude of the waveform at \(\theta = 0\) is significantly reduced for the insulated RVD. The reason for this is that the currents on the RVD arms travel at the speed slower than the speed of wave propagation off the RVD arms. The wavefront at \(\theta = 0\) propagates at the speed of light, but the currents on the RVD arms “drag” the wavefront around the arms, and therefore spread the wavefront around \(\theta = 0\).
Figure 5.4: Radiation pattern of the RVD for a Gaussian pulse with $t_{FWHM} = 0.15\tau_a$. (a) Calculated using the analytical model. (b) Calculated using the numerical model. The interior angle of the RVD is 60°. For the numerical model, $R_0 = 1526\Omega/m$ and the input pulse is incident in a 100Ω transmission line. The relative permittivity of the insulation is 1.5.
5.3 Summary

The time domain operation of a RVD was studied with analytical models. Analytical models were made for RVD using traveling wave elements. One model predicted the radiated field in the plane of a RVD. The radiation pattern of a RVD with interior angle $60^\circ$ was plotted. The optimal interior angle for a RVD was found for a range of Gaussian pulses in terms of peak-to-peak amplitude of the radiated field. The second model predicted the radiated field in the plane of a RVD in an insulation. The radiation pattern of a RVD with interior angle $60^\circ$ was plotted. The relative permittivity of the insulation was $\epsilon_{re} = 1.5$. The radiation pattern for the RVD in the insulation material with $\epsilon_{re} = 1.5$ was different from the radiation pattern for the RVD in free space ($\epsilon_{re} = 1.0$). The shape of the waveforms were different for small $|\theta|$. The amplitude of the waveform at $\theta = 0$ was significantly reduced for the insulated RVD. This will be taken into consideration in the design of a practical RVD in the next chapter.
CHAPTER VI

DESIGN AND REALIZATION OF A RESISTIVE VEE DIPOLE ON A PCB

Various methods have been tried to realize the Wu-King profile. One method involves depositing a resistive material of variable thickness on a dielectric rod [13,62]. With this method, one can build a cylindrical antenna for which the profile was originally developed, but it is difficult to achieve an accurate resistive profile. Another method to implement the profile is to taper a resistive film of constant thickness [20,21]. This method gives a better control over the resistive profile, but the realized structure is mechanically weak and the bonding between the resistive film and the drive point can be problematic. In a third method, one may use a series of discrete resistors that approximates the continuous profile. Maloney and Smith approximated a resistive profile by stacking special high-frequency resistors in series [18,63]. However, this structure is also mechanically fragile.

As an alternative method to realize the resistive profile, we use standard off-the-shelf surface-mount chip resistors [59]. The resistors are mounted on metal strips printed on a dielectric substrate using standard printed circuit board manufacturing technology. The structure is schematically drawn in Figure 6.1. The resulting profile is accurate, easy to build, and mechanically stable. For the design of the RVD, \( R_0 = 1526\Omega/m \) and \( h = 30.6cm \) are chosen as the impedance per unit length at the drive point and the length of a dipole arm. The remaining parameters are discussed in the next sections.
Figure 6.1: Diagram of the RVD implemented on a PCB. The metal strips are printed on a PCB. Then, it is loaded with surface-mount chip resistors.
6.1 Resistor Spacing

To realize the resistive profile with standard chip resistors, Eq. (5.1) is discretized. The arm of length $h$ is divided into a number of sections, and a chip resistor is placed in the center of each section so that the resistance of the section agrees closely with that of Eq. (5.1). Figure 6.2 shows a discrete Wu-King profile with 26 chip resistors for an arm. In the figure, the Wu-King profile is graphed in normalized conductance as a function of normalized distance.

The chip resistors in this chapter are 1mm in width and 1.6mm in length. Thus, a maximum of 191 resistors can be used along each arm. However, to reduce the complexity and cost of the RVD, we will minimize the number of resistors used. To
investigate the effect of the number of resistors, a numerical model of a RVD with \( \alpha = 30^\circ \) is developed using the EIGER code suite. In the model, lumped resistor elements are used to simulate the chip resistors.

Figure 6.3 shows the reflected voltages in the feeding transmission line for three RVDs as functions of time for a Gaussian pulse with \( t_{FWHM}/\tau_a = 0.15 \), where \( \tau_a = h/c \) is the time it takes for light to travel the length of a dipole arm. The three resistive dipoles contain 22, 52, and 94 resistors (11, 26, and 47 resistors for each arm).

In the graphs, the reflected voltages for the resistive dipoles with 52 and 94 resistors look identical; however, the waveform for 22 resistors looks different. The resistive profile discretized by 11 resistors poorly approximates the continuous profile, which is well approximated by 26 and 47 resistors. The discretized resistive profile with 26 resistors is chosen for the RVD, because we want to minimize the number of resistors.

6.2 Substrate Properties

The substrate thickness also affects the performance of the antenna. To investigate the effects of the substrate thickness, the radiated field on boresight of the RVD is plotted for a range of substrate thicknesses in Figure 6.4 when the antenna is excited by a Gaussian pulse with \( t_{FWHM}/\tau_a = 0.15 \) incident through a 100\( \Omega \) transmission line\(^1\). Note that the radiated waveforms as well as their amplitudes depend on the substrate thickness. The reason for this is that the speed of the current on the antenna varies as a function of the substrate thickness.

Figure 6.5 shows the effect of the substrate thickness on the speed of the current.

\(^1\)EIGER cannot efficiently calculate the currents of the RVD on a finite sized substrate. They can be approximately obtained by using an infinite dielectric slab whose thickness is equal to the substrate. However, the radiated fields from the RVD on the slab are dispersed due to the infinite extent of the slab. This problem can be alleviated by first obtaining the currents on the dielectric slab, and then calculating the radiated fields from those currents when they are placed in free space instead of on the slab.
Figure 6.3: Reflected voltages in the time domain for the resistive vee dipoles with 22, 52, and 94 resistors when a Gaussian pulse with $t_{FWHM}/\tau_a = 0.15$ is incident through 100Ω transmission line. The interior angle for the antennas is $2\alpha = 60^\circ$. 
Figure 6.4: Normalized Radiated fields as functions of time for a Gaussian pulse with $t_{FWHM}/\tau_a = 0.15$ incident through a 100Ω transmission line. Each line represents the radiated field on boresight of a RVD with $2\alpha = 60^\circ$ printed on a substrate (FR-4; $\varepsilon_r = 4.2$) whose thickness varies from 0 to 1.6mm. The 0mm-thick substrate is equivalent to the free space.
Figure 6.5: Comparison of currents at a number of points along the arms of the vees in the free space and the vees printed on a 1.6mm-thick FR-4 ($\epsilon_r = 4.2$) substrate when a Gaussian pulse with $t_{FWHM}/\tau_a = 0.15$ is incident through a 100Ω transmission line. The interior angle of the antennas is $2\alpha = 60^\circ$.

In the figure, the currents of the vee in free space and the vee on 1.6mm-thick FR-4 substrate are compared. The currents at a number of points along the arms are plotted as functions of time and vertically offset by the distance from the drive point. The current pulses for both antennas are launched at the same time. As they travel away, the current pulse of the RVD printed on the 1.6mm-thick FR-4 substrate appear later in time than RVD in free space. This indicates that the speed of the current wave is affected by the thickness of the substrate.

The speed of the current wave, $v$, is approximately constant along the antenna
arms and is related to a relative effective permittivity, $\epsilon_{re}$, around the dipole arms as

$$v = \frac{c}{\sqrt{\epsilon_{re}}}$$  \hspace{1cm} (6.1)

The effective relative permittivity depends on the printed strip width as well as the substrate thickness. For a wave launched at the drive point of a RVD, the geometry of the RVD looks roughly like a pair of coplanar strips and the effective relative permittivity of the antenna is approximately that of a pair of coplanar strips:

$$k_1 = \frac{(b - w)}{b},$$

$$k_2 = \frac{\sinh \left( \frac{\pi(b - w)}{2d} \right)}{\sinh \left( \frac{\pi b}{2d} \right)},$$

$$\epsilon_{re} = 1 + \frac{\epsilon_r - 1}{2} \frac{K(k_2^2)K(1 - k_1^2)}{K(k_1^2)K(1 - k_2^2)},$$  \hspace{1cm} (6.2)

where $b$ is the distance from the symmetry plane to the outer edge of the strip lines, $w$ is the width of the printed strips, $d$ is the thickness of the substrate, and $K$ is the complete elliptic integral of the first kind [22,64].

Figure 6.6 shows the effective relative permittivity obtained using Eq. (6.2) for a RVD with $2\alpha = 44^\circ$ on a substrate with $\epsilon_r = 3.4$. For the graph, the parameter $b$ is obtained by first drawing a circle of radius $h/10$ centered at the drive point of the RVD and then taking the arc length from the symmetry plane to the outer edge of a strip line. The effective relative permittivity is plotted as a function of substrate thickness for a number of printed strip widths. The figure shows that the effective relative permittivity increases with increasing substrate thickness and decreases with increasing strip width.

In Figure 6.7, the normalized peak-to-peak amplitudes of the radiated electric fields are plotted as functions of substrate thickness over a range of strip widths using Eq. (6.2) and Eq. (5.19). The amplitude increases with decreasing $d$ and increasing $w$.

To investigate the relation between $\epsilon_{re}$ and the amplitude of the radiated field from the RVD, the simple analytical model developed in Chapter 5 is used to calculate the
Figure 6.6: Effective relative permittivity experienced by a RVD with $2\alpha = 44^\circ$ on a substrate with $\varepsilon_r = 3.4$ as a function of substrate thickness for a number of printed strip widths.
Figure 6.7: Normalized peak-to-peak amplitudes of the radiated electric fields on boresight of a RVD with $2\alpha = 44^\circ$ as functions of substrate thickness over a range of strip widths. The relative permittivity of the substrate is $\epsilon_r = 3.4$. The input pulse to the RVDs is a Gaussian pulse with $t_{FWHM}/\tau_a = 0.15$. The simple analytical model developed in Chapter 5 is used to calculate the amplitudes.
Figure 6.8: Comparison of the peak-to-peak amplitudes of the radiated electric fields of RVDs with $2\alpha = 44^\circ$ over a range of $\epsilon_{re}$. The relative permittivity of the substrate is $\epsilon_r = 3.4$. The source current $I_0(t)$ is a Gaussian pulse with $t_{FWHM}/\tau_a = 0.15$. The simple analytical model developed in Chapter 5 is used to calculate the amplitudes.

Radiated electric fields of RVDs for a range of $\epsilon_{re}$'s. Figure 6.8 shows the peak-to-peak amplitude of the radiated electric field ($|E|_{P-P}$) normalized by $\eta I_0/r$ as a function of observation angle. Note that the amplitude for angles such that $|\theta| < 90^\circ$ strongly depends on $\epsilon_{re}$. The amplitude at $\theta = 0$ drops about 10% for each 0.1 increment of $\epsilon_{re}$. However, for $|\theta| > 90^\circ$, the amplitudes do not vary much.

To decrease $\epsilon_{re}$ or to increase the peak-to-peak amplitude, one has to use a thin substrate with a small $\epsilon_r$ and have wide strips. However, note that the strip width must be chosen such that it is comparable with the width of chip resistors.
Figure 6.9: Realized RVD. The RVD is printed on a vee-shape Kapton film, which is attached to a thick FR-4 plate to enhance mechanical strength. In this figure, the RVD is fed by a double-Y balun.

6.3 Experimental Results

A resistive vee dipole has been designed and realized based on the discussions in the previous sections (Figure 6.9). The arms are printed on a 0.05mm-thick Kapton® substrate ($\epsilon_r = 3.4$ [65]) with interior angle $44^\circ$. Because the substrate is thin and flexible, a thick FR-4 plate is used to support the structure. The width of the strips is 3mm, and 26 chip resistors are soldered along each arm. According to Eq. (6.2), for this design, $\epsilon_{re}$ is about 1.015 at $r/h = 0.1$ as shown in Figure 6.6, which should result in less than a 1.5% reduction in amplitude according to the graph shown in Figure 6.7. The realized vee is represented by dots in Figures 6.6 and 6.7.

Figure 6.10 compares the reflected voltage in the transmission line of the numerical model to that of the experimental model when a Gaussian pulse with $t_{FWHM}/\tau_a =$
Figure 6.10: Comparison of reflected voltages in the 100Ω-feeding transmission line as functions of time for a Gaussian pulse with $t_{FWHM}/\tau_a = 0.15$.

0.15 is incident through a 100Ω transmission line. The experimental result is obtained using the balun assembly described in Figure 3.6. In Figure 6.10, the reflected voltage for the RVD on Kapton agrees well with that for the RVD in free space that is predicted by the numerical model. Thus, the realized RVD works well in terms of the reflected voltage.

The radiation pattern is measured at $r = 2.72m$ away from the drive point of the RVD by a dipole probe whose dimension is shown in Figure 6.11. The dipole probe is 4cm long and connected to a 100Ω-balanced transmission line. In Figure 6.12, the voltage ($V_L$) at the drive point of the dipole probe is plotted as a function of time $t_r/\tau_a$ and vertically offset by angle. In the figure, the thin lines are for the RVD in free space, which are predicted by the numerical model, and the thick lines are for the realized RVD, which are measured. The shapes of the waveform from the measurement agree well with the numerical model. However, the amplitude of the measurement is about 20% larger than that of the numerical model. This amplitude discrepancy is believed to be due to calibration errors and approximations made in the numerical model.

For a RVD to be practical, a balun structure must be included. The results for
Figure 6.11: Dimension of the dipole probe. The dipole arms are essentially the extensions of the center conductors of the semi-rigid coaxial cables. Below the dipole arms, two semi-rigid coaxial cables form a 100Ω-balanced transmission line.

Figure 6.12: Voltages in the 100Ω transmission line connected to a dipole probe. The dipole probe is 2.72m away from the RVD at an angle. The input pulse to the RVDs is a Gaussian pulse with $t_{FWHM}/\tau_a = 0.15$ through a 100Ω transmission line.
two baluns are presented. The first is the balun assembly whose geometry was shown in Figure 3.6 in Chapter 3. The RVDs with the balun assemblies are placed 2.72m apart facing each other as shown Figure 6.13. Figure 6.14 shows the voltage on the balun.

The second is a double-Y balun [66–70] (Figure 6.9) with characteristic impedance $Z_0 = 188\Omega$. The RVDs with a double-Y balun are placed 2.72m apart facing each other. Figure 6.15 shows the voltage on the balun. The Figure 6.15 and Figure 6.14 show the performance of the RVD is deteriorated in terms of the main pulse amplitude and the tail ringing.
Figure 6.15: Received voltages predicted by the numerical model assuming ideal baluns and received voltage measured with double-Y baluns (188Ω) as functions of time for a Gaussian pulse with $t_{FWHM}/\tau_a = 0.15$. The RVDs with the double-Y baluns are facing each other at 2.72m away.

6.4 Conclusion

A practical way to build a resistive vee dipole was proposed and discussed. A RVD was realized on a PCB according to the discussion. The performance of the realized RVD has been investigated through experiments. The results showed that the performance of the RVDs on the PCB is quite good. The received voltages in the baluns have been measured and compared with those with ideal baluns. The baluns significantly degraded the performance of the RVD. Further research should be focused on developing a better balun for use with the RVD.
CHAPTER VII

SUMMARY AND CONCLUSIONS

The objective of this dissertation was to investigate two types of broadband antennas with pulse radiation capability for use in remote sensing applications. The antennas investigated in this dissertation were the impulse-radiating antenna and the resistive vee dipole. The characteristics of these antennas were investigated using numerical, experimental, and analytical models.

First, the IRAs were categorized to lens-type and reflector-type, and the analytical models for the two categories were presented. The parameters needed for the analytical model were presented for typical TEM feed arm structures. Although the lens-type IRA has some advantages over the reflector-type IRA, a lens is generally heavier and more difficult to build than a reflector. Therefore, the reflector-type IRA was selected for further research.

Next, a numerical model for the reflector-type IRA was developed using the EIGER code suite. The IRA was efficiently modeled by removing redundancies in the geometry, such as the orthogonality and reflection symmetry. The performance of the numerical model was validated by comparison with an experimental model. In the experimental model, a small dipole probe was used to measure the radiated fields, and the measured fields were compared to those obtained numerically. The reflected voltage in the feeding transmission line was also measured and compared. The numerical results were in good agreement with the measured results.

Then, the numerical model was used to obtain the responses of a number of IRAs with different geometries. These IRAs had different focal-length-to-diameter ratios ($F/D$), reflectors, and TEM feed arm shapes. The characteristics of these
IRAs were compared in terms of radiated waveforms, reflected voltages in the feeding transmission lines, etc.

Two reflector-type IRAs with $F/D = 0.25$ and $F/D = 0.5$ were compared. With a larger $F/D$, the prepulse was longer and lower as expected in the analytical model. The radiation efficiency at low frequency was higher with a larger $F/D$ because the prepulse was longer. The postpulse amplitude was lower with a larger $F/D$.

Three IRAs with ellipsoidal reflectors were compared with an IRA with a parabolic reflector. The ellipsoidal reflector was parameterized by the distance between the center of the reflector and its closest focus ($F$), the distance between the foci ($\Phi$), and the length of the diameter ($D$). Each IRA had $F/D = 0.5$ in common but different $\Phi/D$’s, e.g., 0.5, 1.0, and 1.5. The apex of the TEM feed structure was located at the focus closer to the reflector. The maximum impulse amplitude and the minimum spot size depended on the pulse rise time as well as $\Phi$, and they did not occur at the optical foci of the reflectors. However, the spot sizes of the IRAs with an ellipsoidal reflector were smaller at distances close to the antenna than the spot sizes of the IRA with a parabolic reflector.

As a way to reduce the aperture blockage in the IRA geometry, offset IRA was considered. Two offset IRAs were modeled. For the first offset IRA, each TEM feed arm was terminated with two resistors, and for the second offset IRA, each TEM feed arm was tapered to one resistor. The reflected voltages in the feeding transmission line were compared, and the comparison showed that the offset IRA with tapered feed arms performed better. The IRA with tapered feed arms was further analyzed and compared with a center-fed IRA. The postpulse of the offset IRA was lower in amplitude and simple in shape than that of the center-fed IRA due to the reduction in the multiple reflections between the TEM feed structure and the reflector. The analysis of the offset IRA showed the asymmetry in the radiated fields due to the asymmetry in the geometry. The offset IRA radiated less power than the center-fed
IRA. However, the offset IRA reflected less power back into the feeding transmission line than the center-fed IRA.

IRAs with a number of terminations were investigated. The terminations of the TEM feed arms were described by a set of curves that has one degree of freedom, which can be used to control the smoothness at the discontinuity. The reflected voltages in the feeding transmission line and the radiated fields on boresight were compared. The best ones in terms of the reflected voltage amplitude and in terms of the postpulse amplitude were selected for each $L$ and compared. The TEM feed arms with $L = F$ with $p < 0$ showed the best performance in terms of the reflected voltage amplitude, and the TEM feed arms with $L = F$ with $p = 0$ showed the best performance in terms of the postpulse amplitude. The shape of the TEM feed arm termination mainly affected the reflected voltage in the feeding transmission line rather than the radiated waveform; however, the shape of the termination had a small effect on the tail of the radiated waveform.

For RVDs, an analytical model was developed using the thin wire approximation. The model was used to predict the radiated fields of an RVD loaded with the Wu-King profile. The model could also be used for RVDs in which the currents propagate at a speed slower than the speed of light in free space, which happens in insulated RVDs and RVDs implemented on PCBs.

Finally, a practical way to build a resistive vee dipole was proposed and discussed. RVDs printed on a circuit board and loaded with standard off-the-shelf surface-mount chip resistors were used to approximate the continuous resistive profile. A study was made to select the number of resistors on the RVD. The effective relative permittivity experienced by the wave propagating along the RVD was related to the permittivity of the PCB, thickness of the PCB, and the width of the printed strip on the PCB. By using this effective relative permittivity with thin wire approximation, the gain of the RVD was found to increase with decreasing effective relative permittivity. A
small effective relative permittivity could be achieved by using a wide strip on a thin PCB with low permittivity.

Based on the initial findings using the approximate theory, a RVD was designed and realized. The performance of the RVD was investigated through experiments. The results showed that the performance of the RVDs on the PCB was quite good. The received voltages in the baluns were measured and compared with those with ideal baluns. The baluns significantly degraded the performance of the RVD. Further research should be focused on developing a better balun for use with the RVD.

In summary, this dissertation has made contributions to the IRA and the RVD. For the IRA, further research should be done on building an experimental model for an offset IRA, and investigating the reduction in the coupling between two offset IRAs placed in the shadows of each other. The TEM feed arms for the experimental model can be terminated by a sharp curve to reduce the reflected voltage in the transmission line. Distributed resistance along the termination instead of a lumped element at the end of the termination can be considered as a matching circuit. For the RVD, a better balun should be developed to obtain stronger and simpler radiation.
APPENDIX A

NUMERICAL MODEL CALIBRATION FOR A DIPOLE SENSOR

In evaluating the performance of EIGER, a dipole was modeled. The reflected voltage in the feeding transmission line was obtained from the EIGER model, and the result was compared with the data measured from an experimental model. The agreement was good, but there was a slight mismatch due to the approximation made to the drive point. Most of this mismatch was removed by a calibration procedure applied to the EIGER model. This chapter discusses this calibration procedure with results.

A.1 Models for Dipole Sensor

Figure A.1 shows the schematic illustration of the reflection measurement from a dipole. Two coaxial cables that are connected to the balun shown in Figure 3.6 are placed in parallel (100Ω). Two wires attached to the center conductors of the coaxial cables form a dipole sensor. The reflection from the dipole (Γ_{dip}) along with the reflections from the three standards (Γ_{open}, Γ_{short}, Γ_{match}) are measured. Then, the reflection coefficient of the dipole sensor is calculated as

\[
\Gamma = \frac{\Gamma_{dip} - S_{11}}{(\Gamma_{dip} - S_{11})S_{22} + S_{12}S_{21}}, \quad (A.1)
\]
Figure A.1: Measurement of reflection coefficients. (a) Measurement of the reflection from the dipole sensor. (b) Measurement of the reflection from three standards.
where $S_{ij}$ are the scattering parameters of the cables and the balun:

$$S_{11} = \Gamma_{\text{match}},$$
$$S_{22} = \frac{2S_{11} - \Gamma_{\text{short}} - \Gamma_{\text{open}}}{\Gamma_{\text{short}} - \Gamma_{\text{open}}},$$
$$S_{12}S_{21} = (\Gamma_{\text{open}} - S_{11})(1 - S_{22}).$$

(A.2)

Figure A.2 shows the mesh for the numerical models. The coaxial cables are included in the mesh for the accuracy of the models. The coaxial cables are modeled as PEC rods. Triangle elements are used to mesh the PEC rods, and wire elements are used to mesh the dipole arms. Two models are shown in the figure. Note the mesh density along the PEC rods. The area close to the dipole arms is densely meshed to model the coupling between the coaxial cables and the dipole arms. The models are differ in the location of the delta-gap feed. For Model I, the delta-gap is located between the center conductor of the coaxial cable and the dipole arm. For Model II, it is located between the coaxial cable mesh and the wire mesh.

The numerical model produces input impedances ($Z_{in}$) seen at the delta-gap feeds. The input impedances are converted to the reflection coefficients:

$$\Gamma_{\text{num,dip}} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0},$$

(A.3)

where $Z_0 = 100\Omega$ is the characteristic impedance of the coaxial cable pair.

The results from the numerical models are compared with those from the experimental model in Figure A.3. The reflected voltages in the 100Ω transmission line are graphed as functions of time. The results from the numerical models agree well with that from the experimental model, but there are slight discrepancies. These discrepancies are believed to be due to the approximations made at the drive point in the numerical model. In Model I, the delta-gap feed drives both the dipole and the center conductors of coaxial cables, which are calibrated out in the experimental model. In Model II, the location of the delta-gap feed is different from the reference point in the experimental model. In the next sections, Model II is used to lessen the
Figure A.2: Mesh and the locations of the delta-gap feeds. (a) Mesh for the numerical models. (b) Location of the delta-gap feed for Model I. (c) Location of the delta-gap feed for Model II.
Figure A.3: Comparisons of the reflected voltages in the 100Ω transmission line as functions of time for a Gaussian pulse with $t_{FWHM} = 0.1\,\text{nsec}$. (a) Comparison of the reflected voltage calculated from Model I and the measured reflected voltage. (b) Comparison of the reflected voltage calculated from Model II and the measured reflected voltage.
Figure A.4: Meshes for the three standards. The numerical models calculates for the input impedances: $Z_{\text{in,open}}$, $Z_{\text{in,short}}$, and $Z_{\text{in,match}}$ for open, short, matched load.

discrepancy between the result from the numerical model and the results from the experimental model.

A.2 Numerical Model Calibration

In the experimental model, the effect of the cables are calibrated out up to the reference point. This procedure can be applied to the numerical model. Figure A.4 shows the meshes for the three standards. The locations of the delta-gap feeds for these meshes are the same as the one for Model II. The numerical models for the three standards generates the input impedances $Z_{\text{in,open}}$, $Z_{\text{in,short}}$, and $Z_{\text{in,match}}$ for open, short, matched load. These are determined at the delta-gap sources, which are located between the PEC rod and the wire. The impedances are converted to the reflection coefficients seen from a transmission line with $Z_0 = 100\,\Omega$:

\begin{align*}
\Gamma_{\text{num,open}} &= \frac{Z_{\text{in,open}} - Z_0}{Z_{\text{in,open}} + Z_0} \\
\Gamma_{\text{num,short}} &= \frac{Z_{\text{in,short}} - Z_0}{Z_{\text{in,short}} + Z_0} \\
\Gamma_{\text{num,match}} &= \frac{Z_{\text{in,match}} - Z_0}{Z_{\text{in,match}} + Z_0}
\end{align*}  

(A.4)
Figure A.5: Comparison of the reflected voltage obtained from the calibrated numerical model and the measured reflected voltage as functions of time for a Gaussian pulse with $t_{FWHM} = 0.1$ nsec.

Then the scattering parameters for the mesh structure below the reference points, which are located between the dipole arms and the center conductor, are found as

\begin{align*}
S_{num,11} &= \Gamma_{num,match}.
S_{num,22} &= \frac{2S_{11} - \Gamma_{num,short} - \Gamma_{num,open}}{\Gamma_{num,short} - \Gamma_{num,open}}, \\
S_{num,12}S_{num,21} &= (\Gamma_{num,open} - S_{11})(1 - S_{22}).
\end{align*}

(A.5)

Using these scattering parameters, the reflection coefficient of the dipole sensor seen at the reference points are obtained as

\begin{equation}
\Gamma_{num} = \frac{\Gamma_{num,dip} - S_{num,11}}{\left(\Gamma_{num,dip} - S_{num,11}\right)S_{num,22} + S_{num,12}S_{num,21}},
\end{equation}

(A.6)

The result from the calibrated numerical model is compared with that from the experimental model in Figure A.5. The reflected voltages in the 100Ω transmission line are graphed as functions of time. The results from the calibrated numerical model agree better with that from the experimental model than the numerical models without the calibration. The performance of the numerical model improved significantly.
A.3 Summary

A calibration procedure that can be applied to a numerical model for a dipole sensor was presented. Without the calibration, the numerical model agreed well with the experimental model, but there was slight discrepancy. With the calibration, the performance of the numerical model improved significantly, and the results from the calibrated model agreed better with the experimental model than the numerical models without the calibration.

The procedure was applied to a dipole sensor. However, this procedure may be applied to a variety of numerical modeling applications.
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Kangwook Kim was born in Mokpo, Korea, in 1972. He received the B.S. degree in electrical engineering from Ajou University, South Korea, in 1997, and the M.S. degree in electrical and computer engineering from the Georgia Institute of Technology, Atlanta, GA in 2001. His research interests include computational electromagnetics, remote sensing, and pulse-radiating antennas.