Measurement of Time-Varying Surface Displacements using a Radar

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Measurement of Time-Varying Surface Displacements using a Radar

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SUMMARY

The objective of this research is to investigate methods for improving the performance of a radar that is intended to be used for a mine detection system that uses elastic and electromagnetic waves simultaneously. Specially, individual antennas and arrays of antennas were investigated with a goal of increasing the standoff distance and the scanning speed of the radar. The antenna configurations under investigation include (1) a focused antenna, (2) a synthetic beamforming array of simple antennas, and (3) an N-element physical array of simple antennas.

First, a basic theory of the focused aperture was studied. Here, properties of the near-field focusing were investigated. The radiation pattern of the aperture was computed to examine the behaviors of the spot size and the sidelobe levels of the patterns. These behaviors were studied as a function of the focal distance and the dimension of the aperture. It was observed that the smallest spot size does not occur at the focal plane when the aperture is placed in the near-field region; it occurs at an intermediate point between the aperture and the focal plane.

For a practical focused antenna, a lens-focused conical corrugated horn antenna was developed and built for use in the measurement of surface displacements. This antenna is more directional so that it can provide a sufficient spatial resolution for the radar. The antenna consists of a conical corrugated horn and a dielectric lens with two foci. This antenna was analyzed theoretically. The analysis included the radiation pattern (one-way pattern) of the antenna and the behavior of the spot size of the pattern. Also, the concept of a two-way pattern was introduced. The two-way pattern was studied because the antenna functions as both a transmitter and
a receiver for the radar. Numerical results for the theoretical model of the antenna showed that as expected, the properties of the radiation pattern of this antenna are similar to those of the focused aperture, and the two-way pattern is seen to be the square of the one-way pattern.

A prototype of the lens-focused corrugated horn was designed and built for use with the radar. The performance of the prototype was also verified with a series of experiments. A modulating scatterer was used to measure the two-way pattern of the antenna. The measured two-way pattern was in good agreement with the theoretically computed pattern. Finally, this antenna was implemented with the land mine detection system and tested in laboratory experiments. Measured results showed that a sufficient spatial resolution can be obtained so that surface displacements can be measured, thus demonstrating the capabilities of the antenna.

Next, a synthetic beamforming array was studied as an alternative technique to obtain the required spatial resolution with the required standoff. A single simple antenna was used for the radar, and it scans to construct a synthetic array. A theoretical model of the synthetic beamforming array was developed. It simulated the signals received by the antenna due to the displacement of the surface, and the beamforming technique was then implemented to reconstruct the surface displacement. Moreover, the received pattern was synthesized for the array to investigate the beamwidth and the sidelobe levels of the pattern. It was found that there is a tradeoff between the beamwidth and the sidelobe levels. In addition, several experiments were performed to investigate the feasibility of the array, and the synthetic beamforming array was used to detect both static and dynamic surface displacements. In both measurements, the spatial resolution of the radar was improved when the beamforming technique was used.

A study was made of a physical array of N antennas for the purpose of improving the scanning speed and creating a greater standoff distance for the radar. Using a simple antenna such as an isotropic or an aperture antenna, two physical array configurations were studied: one-way focus and two-way focus. Theoretical models
were developed for both one-way and two-way focusing arrays. The theoretical models were based on the integral equation method. For both one-way and two-way focusing arrays, appropriate weighting functions to focus the arrays at an arbitrary focal point were chosen, and the received patterns of the arrays were computed to examine the spatial resolution (beamwidth) of the array.

In addition, a two-dimensional finite-difference time-domain (FDTD) model was developed to examine the feasibility of detecting surface displacements and to help to understand part of the above sensing techniques. In this model, a total field/scattered field formulation was used to inject an electromagnetic plane wave, a uniaxial perfectly matched layer was used to truncate the boundaries, and the earth was modeled as a lossy half space. The surface of the earth was modeled to be displaced in the shape of a differentiated Gaussian pulse. These surface displacements were reconstructed by simulating two techniques: (1) a scanning focused antenna and (2) a beamforming array. Numerical results showed that very small surface displacements could be reconstructed successfully using those methods in the FDTD simulation.

Finally, some comparisons were made for the different antenna configurations studied in this work. Comparisons were made in terms of the advantages and the disadvantages of each technique. Important factors such as spatial resolution and scanning speed were explained for each technique.
CHAPTER 1

Introduction

It has been reported that more than 64 countries are contaminated with mines, and there are more than 100 million mines in the ground. As long as these mines are still buried, they will kill and injure thousands of civilians every year and prevent people from accessing land resources. Detecting and removing mines is therefore of international interest, and extensive research efforts have been undertaken to develop more reliable sensing technologies.

Metal detectors have been commonly used worldwide to detect buried mines. However, mines that contain little or no metal (they are made of ceramics and plastics) have been widely produced and buried since the 1970s. Moreover, a lot of metal clutter may exist in mine-fields, resulting in many false alarms. Therefore, it is difficult to use metal detectors to locate mines. This creates a great need for alternate sensing techniques to detect low-metal land mines. Various techniques have been under investigation by various groups to locate and classify low-metal land mines. These techniques include ground-penetrating radars (GPRs), nuclear magnetic resonance, x-rays, infrared detection methods, seismic/elastic methods, etc. In particular, seismic/elastic techniques have been found to be promising for the reliable detection of all types of land mines. The development of an appropriate sensor to sense the seismic/elastic waves is one of the biggest technical challenges to build in such a system. Various sensors can and have been used: laser doppler vibrometers [2][3], radar [4][5], ultrasonic displacement sensors [6], microphone displacement sensors [7][8], ground-
contacting probes such as accelerometers [9], etc. These sensing techniques have some limits; for example, the laser may be negatively affected by ground cover, an ultrasonic sensor may be quite sensitive to environmental noises, and ground-contacting probes may put the operator at risk.

A seismic mine detection system has been under development at Georgia Tech. This system uses a radar-based displacement sensor for the measurement of seismic displacements. The configuration of this system is shown in Fig. 1.1. This system consists of an electromagnetic radar and an elastic wave source. The elastic wave source induces an elastic wave into the earth. The elastic wave displaces the surface of the earth and the mine. The displacement of a mine differs from the earth because of distinct differences in their elastic properties. When a mine is present, the surface of the earth will move differently around the mine than elsewhere because of the waves scattered from the mine. When this occurs, an electromagnetic radar detects the displacement caused by the elastic wave and thus detects the mine.

An electromagnetic radar has been designed and built to measure the displacements of the surface of the earth and the mine. The radar radiates electromagnetic
waves that are reflected off of a vibrating interface. The reflected waves are received by the radar, and the radar scans over discrete positions of the region of interest. A Homodyne system is used to demodulate the received signals. The surface displacements are determined from these demodulated signals. Several issues must be considered to make this radar perform adequately for the mine detection system: (1) high sensitivity to be able to detect very small displacements, (2) sufficiently small spot size, defined by the area on the surface illuminated by the electromagnetic waves, (3) adequate standoff distance for the radar, and (4) selection of operating frequency for the radar. High sensitivity is required for the radar because the maximum amplitude of the elastic surface wave is small and on the order of $10^{-6}$ m. The spot size must be smaller than approximately one half of a wavelength of the elastic wave or of the diameter of the smallest mine; thus, the spatial resolution must be in the range of 2 cm to 5 cm. An adequate standoff distance is also required for the radar. This is obviously necessary in order to account for small-scale topography, to avoid system performance degradation by ground cover such as grass or rocks, and to minimize the risk to the operator. The choice of operating frequency is a compromise; high frequencies may allow a smaller spot size and higher sensitivity for the radar but will also increase the effects of a rough ground surface and of ground cover which may cause degradation of the radar sensitivity and signal attenuation at high frequencies. Thus, an appropriate operating frequency must be selected.

This research mainly focuses on issues of (2) and (3) with given specifications of (1) and (4). Currently, the radar can detect displacements as small as $10^{-9}$ m. To obtain this sensitivity, the radar was designed to minimize the effects of noise, such as the phase noise of the source and the electromagnetic interference from low-frequency magnetic fields [3]. The radar can be operated at frequencies between 2 GHz and 8 GHz; however, an operating frequency of 8 GHz was used to obtain a sufficient spatial resolution. At this frequency, the above required spatial resolution was obtained using an open-ended waveguide or a small horn as the antenna for the radar. However, the antenna must be placed within a few centimeters of the surface
of the earth (standoff distance of 1 or 2 cm); the spatial resolution rapidly decreases as the distance between the antenna and the surface of the earth increases. Another limitation of the current system is that its radar requires several hours to scan 1 m² of ground surface. This scanning time must be reduced if the system is to have practical application.

In this research, several antenna configurations for use in the radar are investigated so that the radar can have an adequate standoff distance while maintaining a sufficient spatial resolution. These antenna configurations are (1) a focused antenna, (2) a synthetic beamforming array of simple antennas, and (3) an N-element physical array of simple antennas. Furthermore, the possibilities of reducing the scanning time for each technique are discussed. Figure 1.2 illustrates an outline of the research and contributions presented in this dissertation. This chapter introduces the current research topic under consideration and defines its limitations. Chapter 2 discusses a focused antenna. The focused antenna is studied to obtain a sufficient spatial resolution at a standoff distance. As preliminary research, a basic theory of the focused aperture is reviewed, and its near-field focusing properties are investigated. The behavior of focused spot size is observed as a function of the antenna parameters. Next, for practical use, a lens-focused corrugated horn antenna is developed. This antenna is theoretically analyzed, designed, and built. Then a series of measurements is shown for validation of the antenna performance.

In Chapter 3, a synthetic beamforming array configuration is studied. A theoretical model is developed. It simulates the signals received by the antenna due to the displacement of the surface, and the beamforming technique is implemented to detect the displacement in fine resolution at a standoff distance. In addition, several experiments are performed to investigate the feasibility of the array.

In Chapter 4, a physical array of N simple antennas is investigated. Here, the array is divided into two main categories: one-way focused and two-way focused arrays. Theoretical models are developed for both configurations. For both arrays, appropriate focusing functions are chosen, and the spatial resolutions of the arrays
Figure 1.2: Outline of research and contributions.
are examined.

Chapter 5 presents a finite-difference time-domain (FDTD) model for detecting surface displacements. Here, a two-dimensional FDTD model is shown to examine the feasibility of detecting surface displacements. This model includes the excitation of the plane wave, a *perfectly matched layer* (PML), the earth modeled as a lossy half space, and the surface displacement of the earth. Very small surface displacements are modeled in the FDTD grid. These surface displacements are then reconstructed using two methods: (1) a scanning focused antenna and (2) a physical array of \(N\) antennas. This model partially helps to understand part of Chapter 2 and of Chapter 4.

In Chapter 6, comparisons are made for the different techniques presented in this dissertation. The comparisons include spatial resolution and the feasibility of the measurement of surface displacements.

Finally, in Chapter 7, the research is summarized, and conclusions are reached. Three sections are appended to this dissertation. Appendix A contains detailed derivations of some equations used in the text. Appendix B presents complete two-dimensional FDTD update equations for the PML absorbing boundary condition with a lossy medium. Appendix C contains details on designing a surface matched lens using horizontal corrugations.
CHAPTER 2

Focused Antenna

The original sensor system (shown in Fig. 1.1) uses an aperture antenna such as a waveguide or a small horn. The spot size is small because the antenna is close to the ground (1 cm to 2 cm above the ground). However, the spot size of the antenna rapidly increases as the distance between the antenna and the surface of the ground increases, resulting in blurred images of the target. This problem can be solved by implementing a focused antenna that directs the electromagnetic waves at a certain position on the surface of the earth [1]. Figure 2.1 shows a configuration of the mine detection system using a focused antenna. Using this focused antenna, a similar spot size is achieved while allowing an adequate standoff distance for the radar. The radar can use either (1) a single focused antenna that scans over the prescribed area or (2) an array of N focused antennas. The latter helps to improve not only the spatial resolution but also the scanning speed by a factor dependent on the number of the antennas that populate the array.

2.1 Focused Aperture

2.1.1 Near-Field Focusing

As preliminary research, a basic theory of a focused aperture is developed. Here, we consider focusing the aperture to a point on its axis near the aperture, which may or may not be in the near-field of the aperture. Figure 2.2 shows the coordinate system of the focused aperture. An arbitrarily shaped planar aperture is placed on the $x$-$y$ plane. Only an electric current source, $J$, is assumed to be present on the aperture
Figure 2.1: Configuration of the mine detection system using a focused antenna (reprinted after [1]).

Figure 2.2: A coordinate system for the focused aperture.
bounded by \( s' \) and is assumed to be polarized in the \( x \)-direction. The vector \( \vec{r} \) is used to indicate the location of the current source, and the position vector \( \vec{R} \) is used to locate the observation point \( P \). The radiated field due to the current source is then expressed as [10]

\[
\vec{E}(\vec{R}) = \frac{-j\omega\mu_0}{4\pi} \int_S \left[ \vec{J}_s e^{-j\kappa r} \right] \nabla' \cdot \nabla' e^{-j\kappa r} \, ds',
\]

(2.1)

where \( \nabla' = \frac{\partial}{\partial x'} \hat{x} + \frac{\partial}{\partial y'} \hat{y} + \frac{\partial}{\partial z'} \hat{z} \). Here, \( x', y', \) and \( z' \) are the coordinate system of the source. This equation is described in detail in Appendix A. If the observation point, \( P \), is located at relatively large distances compared to the dimension of the aperture, \( r \) can be approximated by \( R \), which is a constant. However, \( r \) in the exponential term cannot be approximated as a constant since it is related to the phase shift; \( r \) in the exponential term can be expanded in a series [11]:

\[
r \cong R - \rho \sin \theta \cos(\phi - \beta) + \rho^2 \left[ \frac{1}{2} - \sin^2 \theta \cos^2(\phi - \beta) \right] + \cdots,
\]

(2.2)

where \( \rho = \sqrt{x'^2 + y'^2} \). Assuming paraxial approximation (i.e., very small \( \theta \)), \( \sin^2 \theta \) is then much smaller than unity. Therefore, Eq. (2.2) can be reduced to

\[
r \cong R - \rho \sin \theta \cos(\phi - \beta) + \frac{\rho^2}{2R} \bigg|_{\theta = 0}.
\]

(2.3)

Equation (2.3) indicates that the field integral includes a constant phase shift of \( \frac{x'^2 + y'^2}{2R} \), which is equivalent to a quadratic phase error. Therefore, this term can be eliminated by introducing a complementary phase error on the current source. Thus, the aperture current source with unity amplitude can be selected as [11][12]

\[
\vec{J} = \exp\left[ jk_0 \left( \frac{x'^2 + y'^2}{2R} \right) \right] \hat{z}.
\]

(2.4)

Figure 2.3, for example, shows the quadratic phase distribution of a rectangular aperture source. Eq. (2.4) eliminates the phase error in the field integral so that the aperture source focuses at an axial point, which is located at a distance \( R \) from the origin.
2.1 Focused Aperture

Figure 2.3: Quadratic phase distribution of a rectangular aperture source.

The above derivation and the resulting focal distance $R$ are valid only when $r \sim R$. If the focal point $R$ is too close to the source, the above derivation fails, and the near-field focusing will become inefficient.

On the other hand, the exact phase distribution can be also applied to the current source on the aperture in order to focus the aperture. This exact phase distribution is based on the distance between the location of each current source on the aperture and the focal point:

$$
\tilde{J} = \exp \left[ j k_0 \sqrt{x'^2 + y'^2 + R'^2} \right] \hat{x}.
$$  \hspace{1cm} (2.5)

2.1.2 Focused Spot Size

In this section, the behavior of the focused spot size in the near-field will be investigated. A theoretical model based on the integral method is developed for the focused aperture. Figure 2.4 is a schematic diagram of the focused aperture. It is assumed that the $a \times b$ rectangular aperture is planar and is being taken in the $x$-$y$ plane. The origin of the coordinate system is located at the center of the aperture. Let us assume that the electric field on the aperture bounded by the surface $s'$ is located at the position $\vec{R}'$ and is linearly polarized in the $y$-direction. The electromagnetic field
on the aperture is then
\[ \vec{E}_a(\vec{R}') = A(\vec{R}') e^{-j\psi(\vec{R}') \hat{y}}, \]  
(2.6)
\[ \vec{H}_a(\vec{R}') = -\frac{A(\vec{R}')}{\eta_0} e^{-j\psi(\vec{R}') \hat{x}}, \]  
(2.7)
where \( A(\vec{R}') \) and \( \psi(\vec{R}') \) are the amplitude and phase distributions, respectively. Applying Love’s equivalence theorem, the electric and magnetic surface current densities on the aperture are then obtained from the aperture field:
\[ \vec{J}_s(\vec{R}') = -\frac{1}{\eta_0} A(\vec{R}') e^{-j\psi(\vec{R}') \hat{y}}, \]  
(2.8)
\[ \vec{M}_s(\vec{R}') = A(\vec{R}') e^{-j\psi(\vec{R}') \hat{x}}. \]  
(2.9)
The total radiated electric and magnetic fields at a position vector \( \vec{R} \) on an observation plane can be expressed as [10]
\[ \vec{E}(\vec{R}) = \frac{-j\omega \mu_0}{4\pi} \int \int_{S'} \left[ \vec{J}_s(\vec{R}') + \frac{1}{k_0^2} (\vec{J}_s \cdot \nabla') \nabla' \Psi \right] d\sigma' - \frac{1}{4\pi} \int \int_{S' \times \nabla' \Psi} d\sigma', \]  
(2.10)
\[ \vec{H}(\vec{R}) = \frac{-j\omega \epsilon_0}{4\pi} \int \int_{S'} \left[ \vec{M}_s(\vec{R}') + \frac{1}{k_0^2} (\vec{M}_s \cdot \nabla') \nabla' \Psi \right] d\sigma' + \frac{1}{4\pi} \int \int_{S' \times \nabla' \Psi} d\sigma', \]  
(2.11)
where \( \Psi = \frac{e^{-jk_0r}}{r} \) and \( \nabla' = \frac{\partial}{\partial x'} \hat{x} + \frac{\partial}{\partial y'} \hat{y} + \frac{\partial}{\partial z'} \hat{z} \). Here, \( x', y', \) and \( z' \) are defined by the coordinate system of the source. These equations are given in generalized form. In spherical coordinates, Eqs. (2.10) and (2.11) can be written as
\[ \vec{E}(\vec{R}) = \frac{-j\omega \mu_0}{4\pi} \int \int_{S'} \left\{ \vec{J}_s + \frac{1}{k_0^2} [-k_0^2 (\vec{J}_s \cdot \hat{r}) \hat{r} + \frac{3}{r} (jk_0 + \frac{1}{r}) (\vec{J}_s \cdot \hat{r}) \hat{r} - \frac{\vec{J}_s}{r} (jk_0 + \frac{1}{r})] \right\} \times e^{-jk_0r} \frac{1}{r} d\sigma' - \frac{1}{4\pi} \int \int_{S' \times \nabla' \Psi} d\sigma', \]  
(2.12)
\[ \vec{H}(\vec{R}) = \frac{-j\omega \epsilon_0}{4\pi} \int \int_{S'} \left\{ \vec{M}_s + \frac{1}{k_0^2} [-k_0^2 (\vec{M}_s \cdot \hat{r}) \hat{r} + \frac{3}{r} (jk_0 + \frac{1}{r}) (\vec{M}_s \cdot \hat{r}) \hat{r} - \frac{\vec{M}_s}{r} (jk_0 + \frac{1}{r})] \right\} \times e^{-jk_0r} \frac{1}{r} d\sigma' + \frac{1}{4\pi} \int \int_{S' \times \nabla' \Psi} d\sigma', \]  
(2.13)
Figure 2.4: A schematic diagram of the focused aperture.

where \( \vec{r} = \frac{\vec{R} - \vec{R}'}{|\vec{R} - \vec{R}'|} \) and \( r = |\vec{R} - \vec{R}'| \). The derivations for these equations from Eqs. (2.10) and (2.11) are given in Appendix A.

In order to focus the aperture at the focal point, the phase distribution on the aperture was selected using both the quadratic and the exact phases shown in the previous section. Also, a cosine amplitude distribution on the aperture was chosen to reduce the sidelobe levels of the radiated pattern. The aperture field (Eq. (2.6)) is then given by

\[
\vec{E}_a(\vec{R}') = \cos\left(\frac{\pi x'}{a}\right) \cos\left(\frac{\pi y'}{b}\right) e^{jk_0\left(\frac{1}{2}\vec{R}'^2\right)} \hat{y} \quad \text{Quadratic phase,} \quad (2.14)
\]

\[
\vec{E}_a(\vec{R}') = \cos\left(\frac{\pi x'}{a}\right) \cos\left(\frac{\pi y'}{b}\right) e^{jk_0\sqrt{x'^2+y'^2+F^2}} \hat{y} \quad \text{Exact phase,} \quad (2.15)
\]

where \( F \) is the focal distance of the aperture.

Using the procedure described above, the radiated patterns (one-way patterns) were computed for an operating frequency of 8 GHz. The patterns were obtained from Eq. (2.12) for a square aperture \( a = b = A \) with \( A = 20 \text{ cm} = 5.33\lambda \). Results are shown for three focal lengths, \( F = 10, 20, \) and \( 30 \text{ cm} \), resulting in the \( (A/\lambda)/\sqrt{F/\lambda} \)
Figure 2.5: One-way power pattern ($E$-plane) of the focused aperture with the dimension of 20 cm by 20 cm when $F = 10$ cm: (a) using quadratic phase distribution and (b) using exact phase distribution. Top: pseudo color graphs as a function of $y$ and $h$. Bottom: relative power and 3 dB spot size along height $h$. 
Figure 2.6: Relative power patterns as a function of $y$ when $h = 5$ cm, 10 cm, and 15 cm: (a) using quadratic phase distribution and (b) using exact phase distribution.
Figure 2.7: One-way power pattern (E-plane) of the focused aperture with the dimension of 20 cm by 20 cm when $F = 20$ cm: (a) using quadratic phase distribution and (b) using exact phase distribution. Top: pseudo color graphs as a function of $y$ and $h$. Bottom: relative power and 3 dB spot size along height $h$. 
Figure 2.8: Relative power patterns as a function of $y$ when $h = 10$ cm, 20 cm, and 30 cm: (a) using quadratic phase distribution and (b) using exact phase distribution.
2.1 Focused Aperture

\[ A = 20 \text{ cm and } F = 40 \text{ cm} \]

Figure 2.9: One-way power pattern (\(E\)-plane) of the focused aperture with the dimension of 20 cm by 20 cm when \(F = 40\) cm: (a) using quadratic phase distribution and (b) using exact phase distribution. Top: pseudo color graphs as a function of \(y\) and \(h\). Bottom: relative power and 3 dB spot size along height \(h\).
Figure 2.10: Relative power patterns as a function of $y$ when $h = 20$ cm, 40 cm, and 60 cm: (a) using quadratic phase distribution and (b) using exact phase distribution.
A = 70 cm and F = 35 cm

Figure 2.11: One-way power pattern (E-plane) of the focused aperture with the dimension of 70 cm by 70 cm when \( F = 35 \) cm using exact phase distribution. Left top: pseudo color graphs as a function of \( y \) and \( h \). Left bottom: relative power and 3 dB spot size along height \( h \). Right: relative power patterns as a function of \( y \) when \( h = 25 \) cm, 35 cm, and 45 cm.
Figure 2.12: A contour graph of the 3 dB spot size in centimeters as a function of $F$ and $A$: (a) using quadratic phase and (b) using exact phase.
2.1 Focused Aperture

ratios of 3.27, 2.31, and 1.89, respectively. For each of these focal lengths, a set of graphs is presented in Figs. 2.5-2.10. Pseudo color graphs are shown for the E-plane pattern as a function of $y$ and the location of the observation plane $h$. These graphs are plotted on a 40 dB scale. The power distributions of these pseudo graphs are plotted as a function of $h$ with $y = 0$. Also, the 3 dB spot sizes of the patterns are graphed as a function of $h$. Results are shown for both the quadratic and the exact phase distributions of the aperture. It is seen that the maximum power and the minimum spot size do not appear on the focal plane for either the quadratic or the exact phase; they occur at an intermediate location between the aperture and the focal plane. Also, it is seen that the results for the quadratic phase become similar to those for the exact phase when $F$ increases; the results of the quadratic phase are seen to be almost identical to those of the exact phase when $F = 40$ cm. For both cases (quadratic and exact phases), the locations of the maximum power and the minimum spot size are seen to deviate more from the focal plane as the $(A/\lambda)/\sqrt{F/\lambda}$ ratio decreases.

Figure 2.11 shows the similar results for different aperture parameters: $A = 70$ cm and $F = 35$ cm $(A/\lambda)/\sqrt{F/\lambda} = 6.11)$. In these graphs, the results are shown for only the exact phase case. It is seen that the maximum power and the minimum spot size occur at the location very close to the focal plane. It has been empirically shown that the $(A/\lambda)/\sqrt{F/\lambda}$ ratio must be greater than 5 to focus an aperture at a location near the focal point [12].

In order to observe the behavior of the focused spot size, in Fig. 2.12, the 3 dB spot size is plotted as contours as a function of $A$ and $F$ for the quadratic and the exact phases. The spot size is obtained from the one-way pattern $E$-plane cut when the observation plane is placed at the focal plane $(h = F)$. The spot size is seen to decrease with increasing $A$ and decreasing $F$ for both cases.
2.2 Lens-Focused Corrugated Horn

For a practical focused antenna, a lens-focused conical corrugated horn antenna has been investigated for use in the measurement of surface displacements. A diagram of the antenna system is shown in Fig. 2.13. It consists of a conical corrugated horn and a dielectric bifocal lens. The lens has foci at the surface of the ground and at the open end of a circular waveguide. The lens and the waveguide are connected by the conical corrugated horn. The electromagnetic wave is excited by the horn, propagates through the lens, and is then focused at a point defined by the focal length of the lens.

This antenna was analyzed theoretically. The analysis includes the radiation pattern of the antenna and the behavior of the spot size of the pattern. Also, a concept of the two-way pattern was introduced. The two-way pattern was studied because the antenna functions as both a transmitter and a receiver for the radar. A prototype of the lens-focused corrugated horn has been designed and built and is shown in this section. The performance of the prototype was also verified with a series of experiments.
2.2 Lens-Focused Corrugated Horn

Figure 2.14: A coordinate system of the corrugated horn.

2.2.1 Illuminated Wave by a Corrugated Horn

Figure 2.14 shows the corrugated horn and its coordinate system. The phase center of the horn is placed at the origin, and the horn is confined by a semi-flare angle, \( \theta_e \). The corrugated horn was used instead of a conventional conical horn because the corrugated horn provides equal phase center of the illuminated beam in orthogonal planes (\( E \)- and \( H \)-planes). Also, the field on the horn aperture is tapered toward the edge. The horn is excited by the \( TE_{11} \) mode propagating in the circular waveguide. The \( TE_{11} \) mode is converted into the hybrid \( HE_{11} \) mode by the discontinuity between the circular waveguide and the horn and is facilitated by the corrugations in the horn [13][14]. Assuming the field is linearly polarized, the field in the horn then can be approximately described in terms of a single spherical hybrid mode:

\[
\vec{E}(\theta, \phi, r) = f^1_\nu(\theta)(\cos \phi \hat{\phi} - \sin \phi \hat{\theta}) \frac{e^{-jkr}}{r},
\]

(2.16)

where

\[
f^1_\nu(\theta) = \frac{P^1_\nu(\cos \theta)}{\sin \theta} + \frac{dP^1_\nu(\cos \theta)}{d\theta},
\]

(2.17)

where \( P^1_\nu(\cos \theta) \) is the associated Legendre function of the first kind [15]. \( \nu \) is determined by the solution of the equation:

\[
\frac{P^1_\nu(\cos \theta_e)}{\sin \theta_e} + \frac{dP^1_\nu(\cos \theta_e)}{d\theta_e} = 0.
\]

(2.18)
Table 2.1: First root $\nu$ of $f^1_{\nu}(\theta_e) = 0$ for $HE_{11}$ mode.

<table>
<thead>
<tr>
<th>$\theta_e$ [deg]</th>
<th>$\nu$</th>
<th>$\theta_e$ [deg]</th>
<th>$\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.0</td>
<td>13.3119</td>
<td>21.0</td>
<td>6.1313</td>
</tr>
<tr>
<td>11.0</td>
<td>12.0626</td>
<td>22.0</td>
<td>5.8364</td>
</tr>
<tr>
<td>12.0</td>
<td>11.0221</td>
<td>23.0</td>
<td>5.5674</td>
</tr>
<tr>
<td>13.0</td>
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<td>14.0</td>
<td>9.3885</td>
<td>25.0</td>
<td>5.0949</td>
</tr>
<tr>
<td>15.0</td>
<td>8.7357</td>
<td>26.0</td>
<td>4.8863</td>
</tr>
<tr>
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<tr>
<td>18.0</td>
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<tr>
<td>19.0</td>
<td>6.8152</td>
<td>30.0</td>
<td>4.1931</td>
</tr>
<tr>
<td>20.0</td>
<td>6.4560</td>
<td>31.0</td>
<td>4.0483</td>
</tr>
</tbody>
</table>

$f^1_{\nu}(\theta)$ can be expressed in a more convenient form:

$$f^1_{\nu}(\theta) = \frac{(1 + \nu \cos \theta) P^1_{\nu}(\cos \theta)}{\sin \theta} - \frac{(\nu + 1) P^1_{\nu-1}(\cos \theta)}{\sin \theta}. \quad (2.19)$$

$\nu$ was computed using a numerical method of root finding \cite{16}. Table 2.1 shows values of $\nu$ for the $HE_{11}$ mode in the range of $10^\circ \leq \theta_e \leq 30^\circ$.

Using the values in Table 2.1, the function $f^1_{\nu}(\theta)$ is plotted as a function of $\theta$ for three values of $\theta_e$ in Fig. 2.15. The curves are normalized by $\nu(\nu + 1)$ because the maximum value is $f^1_{\nu}(0) = \nu(\nu + 1)$. When $\theta > \theta_e$, the field distribution oscillates. However, the field distribution at the angle beyond $\theta_e$ is not necessary because the horn is confined by a flare angle, $\theta_e$. Using this field distribution, the field at the aperture of the horn is computed. This field illuminates the surface of the lens, and then, the effect of the lens is analyzed in the following section.

2.2.2 Ray Tracing Technique

Upon obtaining the illumination on the inner surface of the lens, the field propagating through the lens can be obtained using ray tracing technique and geometrical optics (GO) \cite{17}\cite{18}. A schematic diagram of the lens is shown in Fig. 2.16. The dielectric lens with a refraction index, $n$, and a diameter, $D$, has two focal lengths, $F_1$ and $F_2$.
Figure 2.15: Normalized \( f_j^i(\theta) \) pattern for different \( \theta_e \).

(inner and outer focal lengths). The diameter is confined by the semi-flare angle, \( \theta_e \) of the horn. The inner focal point is the same as the phase center of the horn. The inner and outer surfaces are hyperbolic curves. The lens profile is a function of the diameter, the focal length, and the refraction index and is rotationally symmetric. Given those parameters with \( \phi = 0^\circ (y = 0) \), the geometry of the inner surface of the lens can be defined as [13]

\[
x = r \sin \theta, \quad 0 \leq \theta \leq \theta_e, \tag{2.20}
\]

\[
z = \frac{a_1 n + \sqrt{(a_1 n)^2 - (n^2 - 1)(a_1^2 - x^2)}}{n^2 - 1}, \tag{2.21}
\]

where \( a_1 = (n - 1)f_1 \). The geometry of the outer surface of the lens can be given in a similar manner.

Figure 2.17 shows the refraction of the ray at the lens contours. The lens with a refraction index, \( n_2 \), is placed in the air. Incident fields, \( \vec{E}^i \) and \( \vec{H}^i \), in the direction of \( \hat{s}^i \) emanate from the source at the point \( A \). Let us assume that the incident fields at \( B \) on the interface of \( \Omega_1 \) and \( \hat{s}^i \) are known. For example, the incident fields and
\( \hat{s}^i \) can be obtained from Eq. (2.16). The incident fields are partially reflected and transmitted at \( B \). The transmitted fields, \( \hat{E}_1^t \) and \( \hat{H}_1^t \), propagate through within the lens in the direction of \( s_1^t \). Then, \( \hat{H}_1^t \) at the location just before \( C \) can be derived from GO [19]:

\[
\hat{H}_1^t \bigg|_{\text{just before } C} = (DF_1)T_1 e^{-jk_2 BC} \hat{H}_1^t \bigg|_{\text{just before } B},
\]

\[
\hat{E}_1^t \bigg|_{\text{just before } C} = -\eta_2 s_1^t \times \hat{H}_1^t \bigg|_{\text{just before } C},
\]

where \( k_2 \) and \( \eta_2 \) are the wave number and the intrinsic impedance in the medium 2, respectively, \( DF_1 \) is the divergence factor of the surface \( \Omega_1 [19] \), and \( BC \) is the path length between \( B \) and \( C \). The divergence factor is explained in detail in Section 2.2.3.

The second refraction occurs at the interface of \( \Omega_2 \). In a similar manner, the transmitted fields, \( \hat{E}_2^t \) and \( \hat{H}_2^t \), on the interface of \( \Omega_3 \) can be obtained from

\[
\hat{H}_2^t \bigg|_{\text{at } D} = (DF_2)T_2 e^{-jk_1 \delta} \hat{H}_1^t \bigg|_{\text{just before } C},
\]

\[
\hat{E}_2^t \bigg|_{\text{at } D} = -\eta_1 s_2^t \times \hat{H}_2^t \bigg|_{\text{at } D},
\]

where \( k_1 \) and \( \eta_1 \) are the wave number and the intrinsic impedance in the medium 1, respectively, \( DF_2 \) is the divergence factor of the surface \( \Omega_2 \), and \( \delta \) is the path length between \( C \) and \( D \). \( \delta \) is infinitesimal resulting in \( DF_2 \simeq 1 \) and \( e^{-jk_1 \delta} \simeq 1 \).
2.2 Lens-Focused Corrugated Horn

Figure 2.17: Refraction of the ray at the lens interfaces.

$T_1$ and $T_2$ are the Fresnel transmission coefficients at the points $B$ and $C$. These coefficients are divided into two components, a parallel ($T_{||}$) and a perpendicular ($T_{\perp}$) component to the plane of incident [20]:

\begin{align*}
T_{1||} & = \frac{2n_1 \cos \theta_1^i}{n_1 \cos \theta_1^i + n_2 \cos \theta_1^t}, \\
T_{1\perp} & = \frac{2n_1 \cos \theta_1^i}{n_1 \cos \theta_1^i + n_2 \cos \theta_1^t}, \\
T_{2||} & = \frac{2n_2 \cos \theta_2^t}{n_2 \cos \theta_2^t + n_1 \cos \theta_1^t}, \\
T_{2\perp} & = \frac{2n_2 \cos \theta_2^t}{n_2 \cos \theta_2^t + n_1 \cos \theta_2^t}.
\end{align*}

Orthonormal Base Vectors for the Interfaces

In order to complete the above procedure, ray incident vectors, $\mathbf{s}_1^i$ and $\mathbf{s}_2^i$, must be determined. For example, the interface, $\Omega_1$ can be described by [18]

$$\Omega : z_1 = f_1(x,y).$$

(2.30)
2.2 Lens-Focused Corrugated Horn

Then, the unit normal vector $\hat{N}_1$ in the forward direction with respect to the $+z$-direction is

$$\hat{N}_1 = \frac{1}{\Delta}(-f_x \hat{x} - f_y \hat{y} + \hat{z}),$$

(2.31)

where $\Delta = \sqrt{1 + f_x^2 + f_y^2}$, $f_x = \frac{\partial f_1(x, y)}{\partial x}$, and $f_y = \frac{\partial f_1(x, y)}{\partial y}$. Here, vectors $\hat{s}^i$ and $\hat{N}_1$ define the plane of incidence. For the interface $\Omega_1$, an orthonormal base vector is defined by $(\gamma_1, \xi_1, N_1)$. $\hat{\xi}_1$ is chosen to be

$$\hat{\xi}_1 = \hat{N}_1 \times \hat{s}^i.$$  

(2.32)

$\hat{\xi}_1$ is a unit vector normal to the plane of incidence. Then,

$$\hat{\gamma}_1 = \hat{\xi}_1 \times \hat{N}_1.$$  

(2.33)

Using those vectors, $\hat{s}_1^t$ can be determined:

$$\hat{s}_1^t = \hat{\gamma}_1 \sin \theta_1^t + \hat{N}_1 \cos \theta_1^t,$$  

(2.34)

where $\sin \theta_1^t = n_1^{-1} \sin \theta_1^i$. $\hat{s}_2^t$ can be computed in the same manner.

2.2.3 Divergence Factor

The factor $DF_1$ in Eq. (2.22) is known as the divergence factor of the transmitted ray at point $C$ in reference to point $B$. It is determined by

$$DF_1 = \sqrt{\frac{\rho_1 \rho_2}{(\rho_1 + s)(\rho_2 + s)}},$$

(2.35)

where $\rho_1$ and $\rho_2$ are the two principal radii of the curvature of the transmitted wavefront passing through point $B$, and $s$ is the distance between point $B$ and point $C$. The determination of $\rho_1$ and $\rho_2$ is quite complicated. Fortunately, Deschamps [21] and Lee [18],[22] developed a simple method. They approximated the wavefront and interface surface in the neighborhood of the refraction point by second-order equations. This method is described step by step in the following as shown in [18].

Using the same notations as in the previous section, let us define two vectors lying in the tangent plane of the surface at point $B$ as

$$\vec{r}_{1x} = \hat{x} + f_x \hat{z} \quad \quad \quad \vec{r}_{1y} = \hat{y} + f_y \hat{z}.$$  

(2.36)
2.2 Lens-Focused Corrugated Horn

The curvature matrix for the refracted wavefront with respect to the vectors, $\vec{r}_{1x}$ and $\vec{r}_{1y}$ is given by

$$\hat{Q}_{\Omega_1} = \begin{bmatrix} \hat{Q}_{11} & \hat{Q}_{12} \\ \hat{Q}_{21} & \hat{Q}_{22} \end{bmatrix}. \quad (2.37)$$

The entries are given by

$$\hat{Q}_{11} = \frac{e^{2}E' - p F'}{\Delta^{2}}, \quad \hat{Q}_{12} = \frac{p E' - e' F'}{\Delta^{2}},$$
$$\hat{Q}_{21} = \frac{p G' - q' F'}{\Delta^{2}}, \quad \hat{Q}_{22} = \frac{q' E' - p F'}{\Delta^{2}}, \quad (2.38)$$

where

$$\Delta = \left[ 1 + \left( \frac{\partial f_{1}(x,y)}{\partial x} \right)^{2} + \left( \frac{\partial f_{1}(x,y)}{\partial y} \right)^{2} \right]^{1/2}, \quad (2.39)$$
$$E' = 1 + \left( \frac{\partial f_{1}(x,y)}{\partial x} \right)^{2}, \quad (2.40)$$
$$F' = \left( \frac{\partial f_{1}(x,y)}{\partial x} \right) \left( \frac{\partial f_{1}(x,y)}{\partial y} \right), \quad (2.41)$$
$$G' = 1 + \left( \frac{\partial f_{1}(x,y)}{\partial y} \right)^{2}, \quad (2.42)$$
$$e' = -\Delta^{-1} \frac{\partial^{2} f_{1}(x,y)}{\partial x^{2}}, \quad (2.43)$$
$$f' = -\Delta^{-1} \frac{\partial^{2} f_{1}(x,y)}{\partial x \partial y}, \quad (2.44)$$
$$g' = -\Delta^{-1} \frac{\partial^{2} f_{1}(x,y)}{\partial y^{2}}. \quad (2.45)$$

All $x$, $y$, and $z$ coordinates are with respect to point $B$. This curvature matrix is then converted to the matrix in terms of $\hat{\gamma}$ and $\hat{\zeta}$ vectors:

$$Q_{\Omega_1} = M^{-1} \hat{Q}_{\Omega_1} M, \quad (2.46)$$

where

$$M = \begin{bmatrix} \vec{r}_{1x} \cdot \hat{\gamma} & \vec{r}_{1x} \cdot \hat{\zeta} \\ \vec{r}_{1y} \cdot \hat{\gamma} & \vec{r}_{1y} \cdot \hat{\zeta} \end{bmatrix}. \quad (2.47)$$
2.2 Lens-Focused Corrugated Horn

It is assumed that the incident wavefront at point $B$ is a spherical wavefront that has a radius $r$. Then, the curvature matrix of the wavefront can be expressed in terms of $\hat{x}$ and $\hat{y}$:

$$Q_i = r^{-1}I.$$  \hspace{1cm} (2.48)

Then, the curvature matrix of the transmitted wavefront passing through point $B$ can be obtained from the following matrix equation:

$$n_2 B_i^T Q_t B_t = B_i^T Q_i B_i + (n_2 \cos \theta_1^i - \cos \theta_i) Q_{\Omega_i},$$  \hspace{1cm} (2.49)

where

$$B_i = \begin{bmatrix} -\cos \theta_i & 0 \\ 0 & a \end{bmatrix}, \quad B_t = \begin{bmatrix} -\cos \theta_1^i & 0 \\ 0 & a \end{bmatrix}.$$  \hspace{1cm} (2.50)

Upon obtaining $Q_t$ from Eq. (2.49), the principal radii of the transmitted wavefront, $\rho_1$ and $\rho_2$ are given by the roots of the following quadratic equation:

$$\frac{1}{\rho^2} - \frac{1}{\rho} \text{trace } Q_t + \det Q_t = 0.$$  \hspace{1cm} (2.51)

The divergence factor of the transmitted ray at point $C$ in reference to point $B$ is then obtained from Eq. (2.35). $DF_2$ can be obtained in a similar manner. However, due to the infinitesimal distance of $\delta$ in Fig. 2.17, $DF_2$ is approximated to be equal to 1.

2.2.4 One-Way Pattern

Thus far, the fields going in and going out from the outer surface of the lens are found by ray tracing. First order reflections from both inner and outer surfaces are considered in this computation by using the Fresnel reflection coefficient. Then, using the equivalence theorem, the outer surface of the lens can be replaced by an imaginary surface that contains the equivalent electric and magnetic surface current densities ($\vec{J}_s$ and $\vec{M}_s$) on the surface [19]. The imaginary surface must be presumed to be very close to the outer surface of the lens ($\Omega_3$ in Fig. 2.17). The equivalent electric and
magnetic surface current densities can be determined from the electric and magnetic fields exiting from the lens:

$$\mathbf{J}_s = \hat{N}_2 \times \mathbf{H}_3,$$  \hspace{1cm} (2.52)

$$\mathbf{M}_s = -\hat{N}_2 \times \mathbf{E}_3.$$  \hspace{1cm} (2.53)

From these, the radiation pattern of the lens can be found in terms of the lens’s outer surface current densities. The electric and magnetic fields at the position \(\mathbf{R}\) on the observation plane may be represented by the following current-field direct relationships [10]:

$$\tilde{\mathbf{E}}(\mathbf{R}) = \frac{-j\omega \varepsilon_0}{4\pi} \iint_{s'} [\mathbf{J}_s \Psi + \frac{1}{k_0^2} (\mathbf{J}_s \cdot \nabla') \nabla' \Psi] ds' - \frac{1}{4\pi} \iint_{s'} (\mathbf{M}_s \times \nabla' \Psi) ds', \hspace{1cm} (2.54)$$

$$\tilde{\mathbf{H}}(\mathbf{R}) = \frac{-j\omega \varepsilon_0}{4\pi} \iint_{s'} [\mathbf{M}_s \Psi + \frac{1}{k_0^2} (\mathbf{M}_s \cdot \nabla') \nabla' \Psi] ds' + \frac{1}{4\pi} \iint_{s'} (\mathbf{J}_s \times \nabla' \Psi) ds', \hspace{1cm} (2.55)$$

where \(\Psi = \frac{e^{-j k_{0r}}}{r}\) and \(\nabla' = \frac{\partial}{\partial x'} \hat{x} + \frac{\partial}{\partial y'} \hat{y} + \frac{\partial}{\partial z'} \hat{z}\). Here, \(x', y',\) and \(z'\) are defined by the coordinate system of the source. The radiated pattern (one-way pattern) of the antenna is then given by \(|\tilde{\mathbf{E}}(\mathbf{R})|\). This one-way pattern is used to investigate the radiation behavior and the focused spot size when the antenna is placed in the near-zone.

As for the first example, the antenna was chosen with the parameters shown in Table 2.2. The lens has two foci for two surfaces of the lens: the inner focal and the outer focal lengths, \(F_1\) and \(F_2\), respectively. These two foci are assumed to be the same: \(F_1 = F_2 = F\). The antenna is assumed to be placed in the near-field region. Using the above procedure, the one-way power patterns were computed for the antenna. Figure 2.18(a) presents one-way patterns for the \(E\) - and \(H\)-planes as a function of \(x\) and \(y\), respectively, when the observation plane is at the focal plane. It is seen that the sidelobe levels are quite low for both \(E\)- and \(H\)-planes, and the main beams are very similar to each other. Thus, the 3 dB spot sizes of the main beams are seen to be equal for \(E\)- and \(H\)-plane cuts. The pattern behavior can be better
Figure 2.18: One-way power pattern of the lens-focused corrugated horn: (a) pattern of the $E$- and $H$-planes and (b) pseudo color graph of the pattern for the entire region.
Table 2.2: Parameters used for the lens-focused corrugated horn.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inner and outer focal length, $F_1 = F_2 = F$</td>
<td>20 cm</td>
</tr>
<tr>
<td>Lens diameter, $D$</td>
<td>20 cm</td>
</tr>
<tr>
<td>Dielectric constant of the lens, $\varepsilon_r$</td>
<td>2.53</td>
</tr>
<tr>
<td>Operating frequency, $f$</td>
<td>8 GHz</td>
</tr>
<tr>
<td>Polarization</td>
<td>Linear in $x$-direction</td>
</tr>
<tr>
<td>Height of observation plane, $h$</td>
<td>20 cm</td>
</tr>
</tbody>
</table>

understood in Fig. 2.18(b). In this graph, the pseudo color graph of the pattern is plotted for the entire focal plane. The main beam is very clearly visible at the center of the plane, and the pattern is seen to be quite symmetrical for $x$ and $y$ axes.

In order to investigate the focusing characteristics when the antenna is placed in the near-zone, the one-way patterns were computed for a lens with a diameter $D = 20$ cm = $5.33\lambda$, and results are shown for two focal lengths, $F = 10$ and 20 cm, resulting in the $(D/\lambda)\sqrt{F/\lambda}$ ratios of 3.27 and 2.31, respectively. For each of these focal lengths, a set of graphs are presented in Figs. 2.19-2.20. Pseudo color graphs are also shown for the $E$-plane pattern as a function of $x$ and the location of the observation plane $h$. These graphs are plotted on a 40 dB scale. The power distributions of these patterns are plotted as a function of $h$ with $x = 0$. Also, the 3 dB spot sizes of the patterns are graphed as a function of $h$. $E$-plane cuts for several $h$ are also shown as a function of $x$. It is seen that the maximum power and the minimum spot size do not appear on the focal plane for Fig. 2.19(a)(b); they occur at an intermediate point between the aperture and the focal plane. Also, it is seen that the location of the maximum power and the minimum spot size moves toward the lens as the $(D/\lambda)\sqrt{F/\lambda}$ ratio decreases.

Figure 2.21 shows the similar results for different lens parameters: $D = 70$ cm and $F = 35$ cm ($(D/\lambda)\sqrt{F/\lambda} = 6.11$). Unlike the results in Fig. 2.19, it is seen that the maximum power and the minimum spot size occur at the location quite close to the focal plane, and the spot size does not change significantly in the range of 20 cm < $h$ < 37 cm.
Figure 2.19: One-way power pattern (E-plane) of the lens-focused corrugated horn with the lens diameter $D = 20$ cm: (a) $F = 10$ cm and (b) $F = 20$ cm. Top: pseudo color graphs as a function of $x$ and $h$. Bottom: relative power and 3 dB spot size along height $h$. 
Figure 2.20: One-way power pattern ($E$-plane) of the lens-focused corrugated horn with the lens diameter $D = 20$ cm as a function of $x$ with $y = 0$: (a) $F = 10$ cm and (b) $F = 20$ cm.
Figure 2.21: One-way power pattern ($E$-plane) of the lens-focused corrugated horn with the lens diameter $D = 70$ cm and the focal length $F = 35$ cm: Left top: pseudo color graphs as a function of $x$ and $h$. Left bottom: relative power and 3 dB spot size along height $h$. Right: relative power patterns as a function of $x$ when $h = 25$ cm, 35 cm, and 45 cm.
Figure 2.22: A contour graph of the 3 dB spot size (one-way) in centimeters as a function of $F$ and $D$.

In order to observe the behavior of the spot size of the antenna, in Fig. 2.22, the 3 dB spot size is plotted as contours as a function of $D$ and $F$. The spot size is obtained from the $E$-plane pattern when the observation plane is placed at the focal plane ($h = F$). The spot size is seen to decrease with increasing $D$ and decreasing $F$.

The overall focusing properties of the lens are seen to be similar to those of the focused aperture. Similarly in the focused aperture, locating the maximum power and the minimum spot size very close to the focal plane depends on choosing a large enough $(D/\lambda)\sqrt{F/\lambda}$ ratio.

### 2.2.5 Two-Way Pattern

Rather than the one-way pattern of the antenna, it is the two-way pattern involving the reflection from a scatterer back to the antenna that is of interest for the spot size of the antenna when the antenna is implemented with the radar to measure surface displacements. This is because the antenna functions as both the transmitter and the
Figure 2.23: A receiving antenna in the presence of a scatterer.

The two-way pattern can be easily obtained by introducing a scatterer in the presence of the antenna [23]. Figure 2.23 shows an antenna and a scatterer. Let \( I_{in} \) be an input current to the antenna and \( \vec{E}(\vec{R}) \) and \( \vec{H}(\vec{R}) \) be the fields at the position \( \vec{R} \) radiated by the antenna without the scatterer. When the scatterer exists, the antenna receives the fields scattered back from the scatterer. Let the scatterer be a sphere with radius \( a \ll \lambda_0 \), and the sphere is assumed to be located at the position \( \vec{R} \). Then, using the reciprocity theorem, the received open-circuit voltage of the antenna in the presence of the scatterer can be expressed as [23]

\[
V_{oc} = -j\omega\epsilon_0 4\pi a^3 \frac{\epsilon - \epsilon_0}{\epsilon + 2\epsilon_0} \frac{|\vec{E}(\vec{R})|^2}{I_{in}} + j\omega\mu_0 4\pi a^3 \frac{\mu - \mu_0}{\mu + 2\mu_0} \frac{|\vec{H}(\vec{R})|^2}{I_{in}},
\]

(2.56)

which is equivalent to the radar backscattering from the sphere. If the sphere is assumed as a perfect conductor, \( \epsilon = \infty \) and \( \mu = \mu_0 \), resulting in

\[
V_{oc} = -j\omega\epsilon_0 4\pi a^3 \frac{|\vec{E}(\vec{R})|^2}{I_{in}}.
\]

(2.57)

Let the sphere be infinitesimal and be scanned over an arbitrary plane placed \( h \) apart from the lens. Then, the two-way pattern of the antenna is obtained from Eq. (2.57). The shape of the pattern is defined by only non-constant term, \( \vec{E}(\vec{R}) \). Thus, the two-way pattern can be furthermore approximated as \( \alpha |\vec{E}(\vec{R})|^2 \), where \( \alpha \) is a constant that depends on the size of the scatterer and the details of how the antenna is fed.

The equivalent lens with parameters shown in Table 2.2 was chosen, and Eq. (2.57) was used to compute the two-way pattern of the antenna. Figure 2.24 shows the two-
2.2 Lens-Focused Corrugated Horn

Figure 2.24: Two-way patterns of the lens-focused corrugated horn: (a) patterns of the E- and H-planes and (b) pseudo color graph of the two-way pattern.
Figure 2.25: A graph of the 3 dB spot size (two-way) in centimeters as a function of $F$ and $D$.

The two-way pattern of the antenna. The $E$- and $H$-planes are plotted as a function of the position of the infinitesimal sphere in Fig. 2.24(a), and the pseudo color graph of the pattern is shown for the entire plane in Fig. 2.24(b). Since the two-way pattern is the square of the one-way pattern, the sidelobe levels of the two-way pattern are seen to be much lower than those of the one-way pattern, and the 3 dB spot size of the two-way pattern is seen to be narrower.

The overall focusing properties of the lens in the two-way mode are very similar to those of the one-way mode because the two-way pattern is the square of the one-way pattern. The focusing properties of the two-way pattern, therefore, can be analogized from Figs. 2.19-2.21. In Fig. 2.25, a contour graph similar to Fig. 2.22 was duplicated for the two-way pattern. Since the two-way pattern is proportional to the square of the one-way pattern, the spot size is seen to be much smaller than that of the one-way pattern.
2.2 Lens-Focused Corrugated Horn

![Diagram of a corrugated horn](image)

Figure 2.26: Parameters of corrugated horn.

2.2.6 Design of Corrugated Horn

For a prototype to be manufactured, a corrugated horn was carefully designed according to a common design [24]. Figure 2.26 shows a possible configuration for the horn and its parameters. The corrugations that extend circumferentially may be cut normal to the axis of the horn for a small flare angle, $\theta_e$. The corrugations present a capacitive reactance to the passing wave. Surface waves are enforced in the horn if the corrugations are inductive. The depth of corrugations, $d$, must be between $\lambda/4$ and $\lambda/2$. For the best balanced $HE_{11}$ mode, the depth should be a quarter-wavelength; thus, the corrugations must be a quarter-wavelength deep for the slots near the aperture of the horn. The corrugations must be a half-wavelength deep for the slots near the throat of the horn to be a good match for the horn; consequently, the corrugation depths must be tapered. The first slot depth, $d_1$, near the throat is somewhat less than $\lambda_0/2$, and each successive slot decreases in depth over several wavelengths until a depth of $\lambda_0/4$ is achieved. The rest of the slots maintain a depth of $\lambda_0/4$ up to the aperture of the horn. For small flare angle of the horn, it is reported that a pitch, $p$, can be chosen to be $p < \lambda/4$ at the highest frequency for the horn. With a thin wall, $t$, between slots, at least four slots are essential per wavelength along the slant
radius.

To excite the horn, a tapered waveguide transformer can be used to feed the antenna with a standard rectangular waveguide and to transform the rectangular waveguide to a circular waveguide. The cross section of the transformer changes gradually from a rectangular shape to a circular one. The transition section must be made longer than a wavelength so that it allows, in general, a quite satisfactory match. It couples the dominant rectangular mode \((TE_{10})\) and the dominant circular mode \((TE_{11})\).

### 2.2.7 Design of Bifocal Lens

The behavior of the near-field focused spot size of a bifocal lens has been investigated in Sections 2.2.4 and 2.2.5; it has been shown that the smallest focused spot size does not occur at the focal distance. It appears at a distance nearer than the selected focal length. Therefore, if a specific spot size is needed at a desired distance, a parametric study may be required to choose appropriate \(F\) and \(D\). Also, the lens may need to be chosen so that the sidelobe levels of the pattern meet the design specifications.

In order to design a bifocal lens providing appropriate electrical characteristics (e.g., focused spot size and sidelobe levels), a parametric study can be conducted as follows. Let us assume that a corrugated horn is selected to be used with the lens; a phase center and all dimensions of the horn are predetermined. This horn then determines the diameter of the lens to be installed on the aperture of the horn. Also, the horn-lens antenna is assumed to be used at a certain height from the ground for the purpose of mine detection. Assume that a certain focused spot size and appropriately low sidelobe levels at the antenna height are needed. These specifications can be obtained from a parametric study involving the inner focal length, \(F_1\), and the outer focal length, \(F_2\), of the bifocal lens. This is because the lens geometry is a function of \(F_1\) and \(F_2\), accordingly providing different focused patterns as a function of \(F_1\) and \(F_2\).

As an example, a corrugated horn with specific dimensions is selected. Some
Figure 2.27: Contour graphs of the 3 dB spot size in centimeters (a) and the normalized sidelobe levels (b) of the lens-focused corrugated horn antenna with $D = 20$ cm.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter of the horn aperture, $D$</td>
<td>20 cm</td>
</tr>
<tr>
<td>Phase center of the horn from the aperture</td>
<td>23.46 cm</td>
</tr>
<tr>
<td>Semi-flare angle of the horn, $\theta_c$</td>
<td>23.09 degrees</td>
</tr>
<tr>
<td>Diameter of the lens, $D$</td>
<td>20 cm</td>
</tr>
<tr>
<td>Dielectric constant of the lens, $\epsilon_r$</td>
<td>2.53</td>
</tr>
<tr>
<td>Operating frequency, $f$</td>
<td>8 GHz</td>
</tr>
<tr>
<td>Polarization of the excited wave</td>
<td>Linear</td>
</tr>
<tr>
<td>Height of the antenna, $h$</td>
<td>20 cm</td>
</tr>
</tbody>
</table>

important predetermined parameters for the horn and the lens are shown in Table 2.3. With those parameters, the $E$-plane two-way patterns of the antenna (lens-horn) were calculated as a function of the inner focal length, $F_1$, and the outer focal length, $F_2$, of the lens. From these two-way patterns, the 3 dB spot size and the sidelobe level were obtained for different values of $F_1$ and $F_2$. Here, the sidelobe level was obtained by normalizing the total energy confined in the sidelobe level by the total energy confined in the main beam. In this example, the boundary between the main beam and the sidelobe level was taken at the position of $\pm 10$ cm from the center of the main beam. For the lens-horn antenna, the contour graphs of the 3 dB spot size and the sidelobe level as a function of $F_1$ and $F_2$ are shown in Fig. 2.27. The spot size was obtained at a standoff distance of 20 cm. The spot size indicated on each contour is shown in centimeters, and the sidelobe level is dimensionless. It is seen that the spot size decreases in the direction of the arrow, with the smallest spot size appearing on the upper left. However, along the gray line (arrow), the spot size does not change significantly. The sidelobe level behaves in the opposite manner to the spot size. The sidelobe level is seen decreasing in the direction of the arrow. It is seen that there is a tradeoff between the spot size and the sidelobe level. Consequently, an appropriate combination of two foci of the lens must be selected. These graphs were used as a tool to design a bifocal lens to be manufactured. Two foci have been selected at the design point indicated in Fig. 2.27.
Figure 2.28: Photographs of the prototype of the corrugated horn: (a) front view, (b) back view, and (c) side view.
2.2 Lens-Focused Corrugated Horn

Figure 2.29: Photographs of prototypes of lenses: (a) unmatched lens made of Rexolite 1422, (b) unmatched lens made of SL 7510, (c) matched lens made of polycarbonate, and (d) matched lens made of SL 7510.
2.2 Lens-Focused Corrugated Horn

Figure 2.30: Measured real and imaginary parts of the relative dielectric constant of SL 7510.

2.2.8 Prototypes of a Corrugated Horn and Lenses

Four microwave lenses and four corrugated horns have been designed based on the theoretical models shown thus far and subsequently fabricated and assembled.

A corrugated horn was carefully designed and manufactured. A photograph of the prototype of the corrugated horn is shown in Fig. 2.28. The parameters for the horn were selected as shown in Table 2.3. The corrugations in the inner surface of the horn are designed as described in Section 2.2.6. This corrugated horn has been manufactured in a quite different manner than ordinary corrugated horns. It consists of many metal rings that are assembled together to form a corrugated horn. By making it this way, the weight of the antenna can be dramatically reduced. Three more identical corrugated horns were duplicated.

Four different lenses have been designed and fabricated: two ordinary lenses (surface unmatched lenses) and two surface matched lenses. Focal lengths of 20 cm and 30 cm and the diameter of 20 cm were selected for all lenses. The surfaces
of unmatched lenses are relatively easy to fabricate because the geometries of the surfaces of the lenses are simple hyperbolic curves and are rotationally symmetric. Thus, typical machining practices can be used.

One of the ordinary lenses was made of Rexolite 1422 material. This material has a dielectric constant of 2.53 and a loss tangent of approximately 0.001 at 8 GHz. This material maintains a dielectric constant of 2.53 through 500 GHz with a low dissipation factor [25]. For this reason, this material is commonly used for microwave lenses and antennas. Figure 2.29(a) is a photograph of this lens.

A second surface unmatched lens was manufactured and is shown in Fig. 2.29(b). For this work, a “Rapid Prototyping Machine” using stereolithography epoxy (SLA) was used to fabricate a prototype. The SLA machine is available at the School of Mechanical Engineering at Georgia Tech. This machine can make any shaped prototype regardless of the complexity of the geometry. However, this machine can only be used with certain materials, most typically SL 7510. This material belongs to the chemical family of epoxy resin and acrylate ester blend [26]. The electrical properties of this material, such as a dielectric constant and a loss tangent, were measured by placing a sample of the material in a waveguide and measuring the transmission coefficient through the sample. Figure 2.30 shows the measured real and imaginary parts of the dielectric constant, $\epsilon' - j\epsilon''$, for the 8 GHz to 12 GHz frequency range. It is seen that neither part changes significantly over the frequency range. It was found that that the dielectric constant, $\epsilon'$, and the loss tangent, $\epsilon''/\epsilon'$, are approximately 2.94 and 0.0345 at 8 GHz, respectively. Note that this material is significantly more lossy than the Rexolite material.

For the surface matched lenses, a quarter wavelength matching layer is usually used to match the surface of the lens, and several techniques of simulating a quarter wave matching have been reported: corrugated surfaces, arrays of dielectric cylinders, and arrays of holes in the surface of the lens [27]. For this research, corrugated surfaces with horizontal corrugations were chosen. These corrugations are perpendicular with respect to the polarization of the excited $E$-field. Details of the theory and the design
Figure 2.31: Measured real and imaginary parts of the relative dielectric constant of polycarbonate.

of the corrugations are given in Appendix C in this text.

Two prototypes of the surface matched lenses were carefully designed and subsequently fabricated. A manufacturing issue has been taken into account because of the complexities of the corrugation geometries; these lenses would be costly and time consuming to manufacture using typical machining practices. For these lenses, rapid prototyping machines such as a fused deposition modeling (FDM) machine and the SLA machine were used to fabricate the prototypes. One of these lenses was made of the polycarbonate material using the FDM machine. Electrical properties of polycarbonate such as the dielectric constant and the loss tangent have been measured for a sample of this material. The measured real and imaginary parts of the dielectric constant are shown in Fig. 2.31. It is seen that both parts do not significantly change for the 8 GHz to 12 GHz frequency range. It was found that that the dielectric constant and the loss tangent are approximately 2.58 and 0.0039 at 8 GHz, respectively. The polycarbonate material was chosen because this material has a much lower loss
than the SL 7510 material; its loss factor is comparable to that of Rexolite 1422. A photograph of this lens is shown in Fig. 2.29(c). A second surface matched lens was manufactured using the SLA machine. This lens is shown in Fig. 2.29(d).

One of the four lenses and the corrugated horn were assembled together. Figure 2.32 is a photograph of the complete lens-horn antenna manufactured. This antenna consists of a dielectric lens, a corrugated horn, a waveguide transformer, and a waveguide tuner. The lens and the waveguide transformer are connected by the conical metal horn with a corrugated interior. The antenna is fed from a standard rectangular waveguide, and a waveguide transformer is used to transform the rectangular waveguide to the circular waveguide. The cross section of the transformer changes gradually from a rectangular shape to a circular one. Also, a waveguide-type tuner is used in order to match the antenna by achieving the standing wave ratio (SWR) of the antenna desired for the measurement. The waveguide-type tuner consists of a standard rectangular waveguide and three screws inserted into the waveguide. By rotating these screws, a match can be obtained.

2.3 Measurements

The prototype of the lens-focused corrugated horn has been tested with a series of experiments. These experiments include the measurement of the two-way pattern of the antenna and extending the experiment to include time-varying displacements so it can be incorporated into the mine detection system.

All of the measurements have been conducted in a laboratory at the Georgia Institute of Technology with Dr. Waymond Scott, Dr. Gregg Larson, and James Martin. Dr. Waymond Scott designed and built the radar, Dr. Gregg Larson wrote LabView programs to control the radar using a personal computer, and James Martin manufactured most of the hardware needed for the measurements.
Figure 2.32: Photographs of the prototype of the lens-corrugated horn antenna.
2.3 Measurements

Figure 2.33: A block diagram of the Homodyne radar system.

2.3.1 Mono-static Radar

A radar has been designed and built by Dr. Waymond R. Scott to measure the displacements of the surface of the earth and the mine due to the elastic waves [5]. Figure 2.33 is a block diagram of the radar. The antenna radiates electromagnetic waves that are reflected off of a vibrating boundary. Surface displacements due to the vibrating boundary modulate the reflected wave in both amplitude and phase, and the reflected waves are received by the antenna. A Homodyne system is used to demodulate the signals.

Detailed descriptions of the radar are discussed as follows. A signal generator generates a sinusoidal electromagnetic wave source. The generator output is routed to the antenna through a circulator. Here, the circulator is used to help isolate the transmitted and received signals, and a matching network is used to match the impedance of the antenna. The reflected signal from the surface of the earth is received by the antenna. This received signal includes dc-amplitude $A$ and phase $\phi$ and modulated amplitude $\Delta A(t)$ and phase $\Delta \phi(t)$ due to the time-varying surface...
displacements. The received signal is of the form

\[ V_r = [A + \Delta A(t)] \cos[\omega t + \varphi + \Delta \varphi(t)]. \quad (2.58) \]

This received signal is routed, again, by the circulator, into the receiver. The signal is split into two channels: the in-phase channel (I channel) and the quadrature phase channel (Q channel). Both channels are needed to obtain sufficient information to determine the phase modulation \( \varphi \) and \( \Delta \varphi(t) \). The I channel mixes \( V_r \) with the source signal. At the I channel, the output is then

\[ V_I = [A + \Delta A(t)] \cos(\omega t + \varphi + \Delta \varphi(t)) \cdot 2 \cos(\omega t) \]

\[ = [A + \Delta A(t)] \left[ \cos(2\omega t + \varphi + \Delta \varphi(t)) + \cos(\varphi + \Delta \varphi(t)) \right]. \quad (2.59) \]

The high frequency term is filtered out by a low pass filter. Since \( \Delta \varphi(t) \) is very small, \( \cos(\Delta \varphi(t)) \simeq 1 \), and \( \sin(\Delta \varphi(t)) \simeq \Delta \varphi(t) \). Using these approximations, Eq. (2.59) can be rewritten as

\[ V_I = A \cos \varphi + \left[ -A \sin \varphi \Delta \varphi(t) + \Delta A(t) \cos \varphi \right]. \quad (2.60) \]

The Q channel, on the other hand, mixes \( V_r \) with the source signal with a 90° phase shift. At the Q channel, the output can be written as

\[ V_Q = -A \sin \varphi + \left[ -A \cos \varphi \Delta \varphi(t) - \Delta A(t) \sin \varphi \right]. \quad (2.61) \]

Finally, the signals at both channels are sent to the signal processor, which performs functions to integrate and to process the data to determine the displacements of the surface. In order to extract the phase variation, \( \Delta \varphi(t) \), from both channels, an extra procedure is needed:

\[ \alpha \sin \varphi + \beta \cos \varphi = -A \Delta \varphi(t). \quad (2.62) \]

Therefore, the phase variation is then

\[ \Delta \varphi(t) = \frac{-\alpha \sin \varphi - \beta \cos \varphi}{A}. \quad (2.63) \]
Figure 2.34: A schematic diagram of the field measurement using a modulating scatterer.

The amplitude variation, $\Delta A(t)$, can be determined in a similar manner:

$$\Delta A(t) = \alpha \cdot \cos \varphi - \beta \cdot \sin \varphi. \quad (2.64)$$

From these phase and amplitude variations, the surface displacements can be determined.

This radar system was used in most of experiments conducted for this research, and the radar was also used to measure the two-way pattern of the antenna.

2.3.2 Measuring Antenna Pattern using Modulating Scatterers

In order to measure the two-way pattern of the antenna, an experiment was conducted using the method of modulating scatterers. Cullen and Parr [28] and Richmond [29] showed that field distributions can be measured by coherently detecting the back-scattered signal from a modulated dipole, and by using the reciprocity principle, they showed that the back-scattered signal is proportional to the square of the incident electric field at the dipole. Later, the analysis of modulated scatterers was extended
by Harrington’s work [30]. Since then, modulated scatterers have been extensively used to measure electromagnetic fields, including antenna patterns. The choice of a scatterer is mainly dependent on the field distribution to be measured and the required sensitivity. The scatterers are normally relatively small in size compared to a wavelength. The commonly used scatterers are short electric dipoles or small loops.

Figure 2.34 shows an example of the measurement setup to measure the two-way antenna pattern. The setup consists of an antenna under test (AUT), a transmitter and a receiver, a modulating scatterer, and an audio frequency signal source. The scatterer is independently modulated using an audio frequency generated by a modulating source signal. The field incident on a small scatterer is then modulated and back-scattered. This back-scattered signal is then picked up by the AUT and coherently detected by the receiver. This received signal represents the two-way pattern of the AUT because the modulated and back-scattered signal depends on transmission from the transmitter to the scatterer, and then from the scatterer to the receiver.

A half wavelength dipole has been designed and built as the scatterer. A schematic and a photograph of the dipole scatterer are shown in Fig. 2.35. It consists of a printed dipole on a dielectric substrate, a diode switch, and a feeding/decoupling circuit. A microwave PIN switch diode is bridged between the arms of the dipole. Chip resistors are used to limit the current driving the diode and as part of a filter.
Figure 2.36: A schematic diagram and a photograph of the measurement setup.
The two resistors are connected to a chip capacitor forming a low pass filter. The filter provides microwave frequency isolation between the dipole and the feed lines.

This scatterer is then implemented to measure the two-way pattern of the AUT. A schematic diagram and a photograph of the measurement system are illustrated in Fig. 2.36. The measurement was conducted by placing the small dipole in the center of a large anechoic surface. The dipole was driven by an audio frequency of 1 KHz with a peak to peak amplitude of 20 volts.\(^1\) The receiver output was recorded at this frequency using two-second-integration time to build dynamic range above the noise floor. The AUT was then scanned linearly across the surface. The anechoic treatment assured that the measurement could not be contaminated by a signal or by multiple-reflected signals from the ground or from the scatterer.

Using the modulating scatterer method, two-way patterns of the antenna shown in Fig. 2.32 with the four different lenses were measured in a laboratory at the Georgia Institute of Technology. The antenna is placed 20 cm above the modulated dipole scatterer.

**Surface Unmatched Lens Made of Rexolite 1422**

Figure 2.37 shows the two-way patterns measured with four different frequencies for the lens shown in Fig. 2.29(a). For all of frequencies, the \(E\)- and \(H\)-plane patterns are seen to be quite different from each other; the sidelobe levels of the \(H\)-plane pattern are seen to be wider and higher than those of the \(E\)-plane pattern. It was also found that the sidelobe levels are much higher and wider than predicted theoretically. Furthermore, the sidelobe levels are seen to change more than expected with changes in frequency.

These anomalies for the sidelobe levels are thought to happen because of unwanted resonance within the lens; the lens is seen to act as a waveguide resonator that allows resonant fields to flow along the lateral direction inside the lens. The resonance

\(^1\)The level of the driving voltage of the dipole must be several volts. At lower drive levels, the modulation of the dipole was not consistent. This is believed to be due to self-biasing of the diode when it was exposed to strong incident fields.
Figure 2.37: Measured two-way patterns of the surface unmatched lens made of Rexolite 1422: (a) E- and (b) H-planes.
Figure 2.38: Measured two-way patterns of the surface unmatched lens made of SL 7510: (a) $E$- and (b) $H$-planes.
is believed to happen mainly because of the impedance mismatch between the lens and the air. The impedance mismatch of the lens allows multiple reflections at lens interfaces. Multiple reflections from the interfaces degrade the antenna performance due to increases in sidelobe levels.

**Surface Unmatched Lens Made of SL 7510**

Figure 2.38 shows graphs of the two-way patterns measured for the lens shown in Fig. 2.29(b). When compared to Fig. 2.37, the sidelobe level behaves in a quite different manner; the sidelobe levels are lower and narrower than those in Fig. 2.37. The reason for this is believed to be the increased loss in this lens. The increased loss damps the resonance and decreases its effects.

**Surface Matched Lens Made of Polycarbonate**

Two-way patterns have been measured for the surface matched lens shown in Fig. 2.29(c). Figure 2.39 shows the two-way patterns measured with four different frequencies. When compared to Fig. 2.37, the substantial reduction of the sidelobe levels is shown to be obtained with the surface matched lens. This is due to the surface matching layer, yielding the reduction of the multiple reflections within the lens.

**Surface Matched Lens Made of SL 7510**

Figure 2.40 shows the two-way patterns measured for the lens shown in Fig. 2.29(d). When compared to Fig. 2.38, the sidelobe levels are seen to be substantially reduced. Furthermore, the patterns are seen to be consistent with small variations in the frequency.

From the resulting patterns in Figs. 2.39 and 2.40, it is seen that the lens made of SL 7510 performs slightly better than the one made of polycarbonate. The reason for this is that both the loss in the material and the surface matching layer contribute to the reduction of the resonance within the lens made of SL 7510. However, the maximum power received by lenses made of the low loss materials is greater by
Figure 2.39: Measured two-way patterns of the surface matched lens made of polycarbonate: (a) $E$- and (b) $H$-planes.
Figure 2.40: Measured two-way patterns of the surface matched lens made of SL 7510: (a) $E$- and (b) $H$-planes.
approximately 3 dB than that received by lenses made of SL 7510. Therefore, there must be a tradeoff between the maximum power received and the performance of the lens. Table 2.4 shows the absolute maximum power received by each lens.

For comparisons, the measured and calculated patterns are plotted together in Figs. 2.41 and 2.42. These comparisons are shown for the surface matched lenses made of polycarbonate and SL 7510. The calculated patterns were obtained from Section 2.2.5 ignoring the reflections from the lens surfaces and the surface matching layer. In general, the agreement is seen to be good for both planes; the first sidelobe levels and the overall shapes of the patterns are in good agreement. However, for the polycarbonate lens, slight disagreement appears; the first sidelobe levels are seen to be slightly wider than the theoretical results. This disagreement is believed to mainly present because either the dielectric constant of a sample of polycarbonate is not sufficiently accurate², or the resonance has not been completely eliminated due to the low loss factor as discussed earlier.

A two-dimensional two-way pattern has been also measured. This measurement was performed only for surface matched lens made of SL 7510. Figure 2.43 is pseudo color graphs of the measured and calculated two-dimensional patterns for the entire region of interest. These graphs are shown on a 120 dB scale. The measured pattern is seen to be in good agreement with the calculated pattern over the entire region.

²Note that the measured values of the dielectric constant for the polycarbonate sample made using the FDM machine is lower than that published for bulk polycarbonate. This is believed to be due to small amounts of air trapped within the material because of the way the sample is fabricated in the FDM machine. It is not known how consistent the dielectric properties of this material are as a function of location within the lens.
Figure 2.41: Comparisons between the calculated and measured patterns for the surface matched lens made of polycarbonate at 8 GHz: (a) E- and (b) H-planes.
Figure 2.42: Comparisons between the calculated and measured patterns for the surface matched lens made of SL 7510 at 8 GHz: (a) $E$- and (b) $H$-planes.
Figure 2.43: Pseudo color graphs of two-dimensional two-way patterns: (a) measurement and (b) theory.
2.4 Dual Radar Consisting of Two Lens-Focused Corrugated Horns

The radar can use either (1) a single focused antenna (single radar) or (2) an array of N focused antennas (multi radar). The latter increases the scanning speed by a factor dependent on the number of the antennas that populate the array. A schematic diagram of a multi radar consisting of an array of N focused antennas is shown in Fig. 2.44. The focused N-antenna array is placed at height \( h \) and then scans over the area of interest. Each antenna is operated at a slightly different frequency to minimize interference among the antennas. Ideally, each of the N antennas radiates electromagnetic waves and simultaneously receives reflected signals from the surface of the earth.

For this research, a dual radar consisting of an array of two focused antennas has been developed. Figure 2.45 is a photograph of the dual radar. An identical corrugated horn shown in Fig 2.32 has been duplicated. The two surface matched lenses shown in Fig. 2.29(c)(d) were mounted on the corrugated horns. Two new sets of radar have been designed and built by Dr. Waymond Scott. These radars are
2.4 Dual Radar Consisting of Two Lens-Focused Corrugated Horns

Figure 2.45: A photograph of the dual radar.

similar to the radar shown in Fig. 2.33. Installation hardware has been manufactured by James Martin.

2.4.1 Measuring Two-Way Patterns using Dual Radar

To examine the performance of the dual radar, a series of experiments has been conducted. Two-way patterns have been measured using the modulating scatterer method as described in Section 2.3.2. Figure 2.46 is a photograph of the dual radar installed for the measurements of two-way patterns. The setup for this measurement is the same as the previous measurement setup (Fig. 2.36) except for the dual radar. The dual radar consists of two radar sets and two lens-focused corrugated horn antennas. Radar A has the antenna with the surface matched lens made of SL 7510 material and with an operating frequency of 7.9 GHz, and radar B has the antenna with the surface matched lens made of polycarbonate and with an operating frequency of 7.8 GHz. The antenna of radar A is aligned to have its polarization 90° with respect to the polarization of the antenna of radar B (cross polarized); the polarizations of radar A and radar B are parallel to the x- and y-axes, respectively.
Figure 2.46: A photograph of dual radar installed for the measurements of two-way patterns.

This will minimize interference that may occur between these two radars.\textsuperscript{3} The dipole scatterer is oriented to 45° with respect to polarizations of both radars. The dual radar was scanned over the dipole scatterer, and the output of the radars was recorded. This was done in two separate scans with the output of radar $A$ recorded in the first scan (scan $A$) and the output of radar $B$ recorded in the second scan (scan $B$). Both radars were on during both scans.

The measured patterns for both scans are shown in Fig. 2.47 for both $E$- and $H$-planes. When compared to Figs. 2.40 and 2.39 (measurements with a single radar), it is seen that the resulting patterns are almost identical for the single radar and the dual radar. From these results, it can be projected that the dual radar can be implemented with the mine detection system without any serious problem. The dual radar will allow not only a standoff distance but also improvement of the scanning speed by a factor of 2.

\textsuperscript{3}Actually, interference (cross-talk) should not matter; each radar is operated at a different frequency, and the radar filters out the unwanted signal at the intermodulation frequency (IF) step.
Figure 2.47: Measured two-way patterns of the dual radar: (a) scan A and (b) scan B. The origin in each of these graphs is the position where either antenna A or antenna B is directly above the scatterer.
2.4 Dual Radar Consisting of Two Lens-Focused Corrugated Horns

Figure 2.48: Laboratory experimental model to detect land mines using dual radar.

2.4.2 Measuring Time-Varying Surface Displacements using Dual Radar

The dual radar was then tested with the help of Dr. Gregg Larson in the laboratory model that was used for the development of the mine detection system. This model is shown in Fig. 2.48. The model consists of a sandbox and the radar, which is scanned over the surface of the sand. The scanned area is 120 cm by 80 cm. A TS-50 anti-personnel mine with a diameter of 8 cm and height of 3 cm is buried at the center of the area. The top of the mine is 1 cm below of the surface of the sand. An elastic wave transducer is placed on the surface of the sand and used to launch elastic waves into the sand. These elastic waves travel across the surface of the sand and cause the surface of the sand and the mine to be displaced. The dual radar is used to measure the displacement of the surface of the sand and thus detect the mine. The response of the radar is measured on a grid of the discrete positions in the scanned region. The grid consists of $121 \times 41$ spaced points that are spaced 1 cm and 2 cm apart in the $x$- and $y$-directions, respectively. The output of the radar is measured as a function
of time at each of these positions. The displacements of the surface of the sand are calculated from these measurements. The dual radar was scanned over the simulated mine field. The measurements were performed in the two scans (scans $A$ and $B$) as described above. In both scans, the antenna is placed at a standoff distance of 20 cm above the surface of the sand.

For both cases, pseudo color graphs of the amplitude of the displacement of the surface of the sand are shown in Figs. 2.49 and 2.50 for scan $A$ and scan $B$, respectively. These graphs are plotted for the entire scanned region at four instants in time on a 40 dB scale. In these graphs, an elastic wave transducer is placed at the bottom of each plot. The elastic waves are traveling across the surface of the sand from the bottom to the top. A TS-50 mine is buried at the center of the scanned region as indicated in each graph. At $T_1$, the elastic waves are traveling toward the mine. At $T_2$, the waves have reached the mine and have interacted with the mine. At $T_3$, the waves are passing the mine and are about to exit the mine. At $T_4$, the waves have passed the mine, and the mine is clearly visible at the center of the scanned region because of a resonance that occurs at the mine location. However, the mine appears slightly bigger than its actual size; this is expected because the antenna averages the displacement over the spot of the antenna. The results of scan $A$ are almost identical with those of scan $B$ except for more noise appearing in the resulting graphs of scan $A$. The differences in noise are believed to be due to differences in the ambient noise level in the laboratory for each of the measurements.

From these results, it can be seen that simultaneously operating two radar systems can yield almost identical results to those shown in Figs. 2.49 and 2.50. At the same time, the two radar systems will halve scanning time.
Figure 2.49: Pseudo color graphs of the displacement of the surface at four instants in time: scan $A$. 
Figure 2.50: Pseudo color graphs of the displacement of the surface at four instants in time: scan B.
CHAPTER 3

Synthetic Beamforming Array

3.1 Introduction

A synthetic beamforming array has been studied as an alternative technique to obtain the required spatial resolution with the required standoff, and some of the results have been published [31][32]. In this array, a single simple antenna such as a waveguide is used. A schematic diagram of this array configuration is shown in Figs. 3.1 and 3.2. As shown in Fig. 3.1, a single antenna is placed at height $h$ above the ground surface, and the antenna is scanned to construct a synthetic array. The antenna is scanned on a uniform squared grid of discrete positions that are spaced $d$ apart. At each discrete position, the antenna transmits electromagnetic waves and receives the waves reflected off the boundary between the air and the earth. After obtaining the received signal at each antenna position, a synthetic array is constructed in order to beamform the array as shown in Fig. 3.2. In other words, the received signals of the antennas are summed with an appropriate weighting function to beamform the array at an arbitrary focal point so that sufficient spatial resolution can be obtained.

This chapter describes a theoretical model of the synthetic beamforming array. This model simulates the signals received by the antenna due to the displacement of the surface, and the beamforming technique is then implemented to reconstruct the surface displacement. In addition, several experiments are performed to investigate the feasibility of the array.
Figure 3.1: A single antenna being scanned over the surface of the earth.

Figure 3.2: A synthetic beamforming array configuration.
3.2 Theoretical Model

A theoretical model of the synthetic array of simple antenna has been developed. The theoretical model is based on the integral equation method. It simulates the signals received by the antenna due to the displacement of the surface, and a beamforming technique is implemented to reconstruct the surface displacements.

In order to study the synthetic beamforming array, a schematic diagram of the theoretical model is shown in Fig. 3.3. The array consists of \( I \times J \) antennas that are placed at height \( h \) above an observation plane and parallel to the \( x-y \) axis. All antennas are assumed to be identical. A small conducting region is scanned over the observation plane.

For the \( i^{th} \) and \( j^{th} \) antenna, let us assume that electric and magnetic current densities \( \vec{J}_i \) and \( \vec{M}_i \) on the aperture of the antenna bounded by the surface \( s' \) are known and are located at the position \( \vec{R}' \). The position vector \( \vec{R} \) is used to locate the
3.2 Theoretical Model

$M \times N$ discrete positions of the small conducting region scanned on the observation plane. The electric fields radiated by the aperture can be expressed as [10]

$$
\tilde{E}^t_{e,ij}(\vec{R}) = \frac{-j\omega \mu_0}{4\pi} \int_{s'} \left[ \tilde{J}_s \Psi + \frac{1}{k_0^2} (\tilde{J}_s \cdot \nabla') \nabla' \Psi \right] ds',
$$

(3.1)

$$
\tilde{E}^t_{m,ij}(\vec{R}) = \frac{1}{4\pi} \int_{s'} (\tilde{M}_s \times \nabla' \Psi) ds',
$$

(3.2)

where $\Psi = \frac{e^{-jk_0 r}}{r}$ and $\nabla' = \frac{\partial}{\partial x'} \hat{x} + \frac{\partial}{\partial y'} \hat{y} + \frac{\partial}{\partial z'} \hat{z}$. Here, $x'$, $y'$, and $z'$ are defined by the coordinate system of the source. The total electric field is then

$$
\tilde{E}^t_{ij}(\vec{R}) = \tilde{E}^t_{e,ij}(\vec{R}) + \tilde{E}^t_{m,ij}(\vec{R}).
$$

(3.3)

Assuming the small conducting region is in the far-field of the source generating $\tilde{E}^t_{ij}(\vec{R})$, the magnetic field is then

$$
\tilde{H}^t_{ij}(\vec{R}) = \frac{1}{\eta_0} \hat{s} \times \tilde{E}^t_{ij}(\vec{R}),
$$

(3.4)

where $\hat{s} = \frac{\vec{r}}{r}$ (radial unit vector). This is valid when the aperture dimension of the antenna is relatively small compared to height $h$. Next, using the equivalence principle, the surface current density on the small conducting region is obtained from the tangential components of the electric and magnetic fields on the surface of the conducting region. Since the surface of the conducting region is approximately planar, the surface current densities are approximated by

$$
\tilde{J}^{eq}_{ij}(\vec{R}) = -\hat{z} \times \tilde{H}^t_{ij}(\vec{R}),
$$

(3.5)

$$
\tilde{M}^{eq}_{ij}(\vec{R}) = \hat{z} \times \tilde{E}^t_{ij}(\vec{R}).
$$

Let $\Delta \tilde{E}^t_{ij}(\vec{R})$ be the electric field at the center of the $i^{th}$ and $j^{th}$ antenna due to these current elements at the location $\vec{R}$. $\Delta \tilde{E}^t_{ij}(\vec{R})$ can be obtained from Eqs. (3.1)-(3.3) with Eq. (3.5). Then, the voltage received by the $i^{th}$ and $j^{th}$ antenna is given by

$$
V^T_{ij} = \sum_{m=1}^{M} \sum_{n=1}^{N} \Delta \tilde{E}^t_{ij}(\vec{R}) \cdot \hat{r}_{ij}(\vec{R}),
$$

(3.6)
where \( \tilde{l}_{ij}^{e}(\vec{R}) \) is the vector effective height (VEH). Here, the VEH is used to determine the voltage induced on the open-circuit terminals of the antenna from the received field. The VEH is determined from the radiated field (Eq. (3.3)) of the antenna [33]:

\[
\tilde{l}_{ij}^{e}(\vec{R}) = \alpha \frac{4\pi |\vec{R}| E_{ij}^{t}(\vec{R})}{-j\eta_0 k_0 e^{-j k_0 |\vec{R}|}}, \tag{3.7}
\]

where \( \alpha \) is a constant that depends on the details of how the antenna is fed. The VEH is a far-field quantity, thus, the observation plane is assumed to be placed in the far-field region of the antenna. The above step must be repeated for each antenna to synthesize the array.

### 3.3 Beamforming Algorithm

After obtaining the received voltage at each antenna position, a beamforming algorithm can be applied. A schematic diagram of the synthetic beamforming array is shown in Fig. 3.4. The configuration of the array is the same as in Fig. 3.3 except for the coordinate system. The location of the antenna is indicated by \( \vec{R}' \), and the
3.3 Beamforming Algorithm

Figure 3.5: Gaussian window function: (a) 3-D graph with $\tau = 7.5\lambda$ and (b) $y$-cut for different value of $\tau$.

The received voltage at each antenna position is known. The array is focused at an arbitrary focal point located at the position $\vec{R}_f$. The received voltages of the antennas are summed with an appropriate weighting function to focus the array at the focal point. The phase of the weighting function is chosen so that the voltages will add constructively by using the conjugate-field matching concept [34][35]. The amplitude of the weighting function is chosen to control the sidelobe levels of the array. The output of the array is then

$$V_A(\vec{R}_f) = \sum_{i=1}^{I} \sum_{j=1}^{J} V_{ij}^r(\vec{R}_f') W(\vec{R}_f, \vec{R}_f') e^{j\alpha_{ij}(\vec{R}_f)}, \quad (3.8)$$

where $W$ is a window function used to adjust the amplitude of the weighting function, and $\alpha_{ij}(\vec{R}_f) = (-1)\{\text{phase of } \Delta V_{ij}^r(\vec{R}_f)\}$, where $\Delta V_{ij}^r(\vec{R}_f) = \Delta \vec{E}_{ij}^r(\vec{R}_f) \cdot \vec{e}_{ij}(\vec{R}_f)$. We have determined empirically that a Gaussian window function performs well:

$$W(\vec{R}_f, \vec{R}_f') = \exp\left(-\frac{1}{(0.6\tau)^2}\right) |\vec{e} \times (\vec{R}_f - \vec{R}_f')|^2, \quad (3.9)$$
where \( \tau \) is a parameter used to adjust the width of the window function at the 50 \% drop point from the peak.\(^1\) The window function is at maximum directly above \( \vec{R}_f, |\hat{z} \times (\vec{R}_f - \vec{R})| = 0 \), and it decreases with increasing distances from this point. This window function is shown in Fig. 3.5. The amplitude of the window function is normalized, and its maximum amplitude is 1. The shape of the function is seen to be symmetric for \( x \)- and \( y \)-axes. The width of the function at a 50 \% drop point from the peak increases with increasing value of \( \tau \).

### 3.4 Synthetic Beamforming Array Pattern

An open-ended rectangular waveguide or a Pyramidal horn was used as the antenna for the synthetic beamforming array. In order to investigate the radiated patterns (one-way patterns) of these antennas, using Eq.(3.3), one-way patterns were computed on the observation plane. In order to use Eq.(3.3), the field distributions over the aperture of the antenna must be known. For an open-ended waveguide, which has the dominant mode of \( TE_{10} \), assuming the \( E \)-field is polarized in the \( y \)-direction over the aperture of the horn, the tangential components of the electromagnetic fields over the aperture can be approximately expressed as [33]

\[
E_y = E_0 \cos\left(\frac{\pi}{a} x'\right), \tag{3.10}
\]

\[
H_x = -\frac{E_0}{Z_{TE}} \cos\left(\frac{\pi}{a} x'\right), \tag{3.11}
\]

where \( Z_{TE} = \eta_0/\sqrt{1 - (\lambda_0/2a)^2} \). The equivalent electric and magnetic current densities over the aperture are then

\[
J_y = -\frac{E_0}{Z_{TE}} \cos\left(\frac{\pi}{a} x'\right), \tag{3.12}
\]

\[
M_x = E_0 \cos\left(\frac{\pi}{a} x'\right), \tag{3.13}
\]

\(^1\)The constant of 0.6 in Eq. (3.9) was empirically determined so that \( \tau \) is the width of the window function at the 50 percent drop point from the peak.
3.4 Synthetic Beamforming Array Pattern

Figure 3.6: $E$- and $H$-plane patterns of the open-ended rectangular waveguide: (a) $h = 20$ cm and (b) $h = 40$ cm.

Figure 3.7: $E$- and $H$-plane patterns of the Pyramidal horn: (a) $h = 20$ cm and (b) $h = 40$ cm.
where $E_0$ is a constant, $\eta_0$ is the impedance in the free space, $x'$ is defined by the coordinate system of the source over the aperture, and $a$ is the aperture dimension of the waveguide along the $x$-direction.

For a Pyramidal horn,

$$E_y = E_0 \cos\left( \frac{\pi x'}{a} \right) e^{-j[k_0(x'^2 + y'^2)/\rho_1]^2}, \quad (3.14)$$

$$H_x = -\frac{E_0}{\eta_0} \cos\left( \frac{\pi x'}{a} \right) e^{-j[k_0(x'^2 + y'^2)/\rho_1]^2}, \quad (3.15)$$

where $x'$ and $y'$ are defined by the coordinate system of the source over the aperture, $a$ is the aperture dimension of the horn along the $x$-direction, and $\rho_1$ and $\rho_2$ are the flare lengths for the $E$- and $H$-plane, respectively. The equivalent electric and magnetic current densities over the aperture are then

$$J_y = -\frac{E_0}{\eta_0} \cos\left( \frac{\pi x'}{a} \right) e^{-j[k_0(x'^2 + y'^2)/\rho_1]^2}, \quad (3.16)$$

$$M_x = E_0 \cos\left( \frac{\pi x'}{a} \right) e^{-j[k_0(x'^2 + y'^2)/\rho_1]^2}. \quad (3.17)$$

The aperture dimension of the waveguide is 2.3 cm by 1 cm, and that of the horn is 5 cm by 5 cm. The antenna is placed at a standoff distance of 20 cm or 40 cm above the observation plane. $E$- and $H$-plane patterns are shown for the waveguide and the horn in Figs. 3.6 and 3.7, respectively. For both antennas, it is seen that the shape of the $E$-plane is broader than that of the $H$-plane, and the overall power level increases with increasing antenna height, and the overall beam shape of the horn is seen to be narrower than that of the waveguide.

With these patterns, the beamforming array pattern is then synthesized using the beamforming technique described earlier. The parameters used in this example are shown in Table 3.1. The array consists of $129 \times 129$ equally spaced antennas with a spacing of $1/4 \lambda$; the array is formed by scanning a single antenna over a grid of $129 \times 129$ equally spaced discrete points. The antenna is placed at a standoff distance, $h$, of 20 cm or 40 cm above the observation plane. The array pattern is obtained from
Figure 3.8: Synthesized pattern of the beamforming array using the open-ended waveguide for different values of $\tau$ when $h = 20\,\text{cm}$ and $h = 40\,\text{cm}$.
Figure 3.9: $E$-plane cuts of the synthesized pattern of the open-ended waveguide for different values of $\tau$ when (a) $h = 20$ cm and (b) $h = 40$ cm.
Figure 3.10: Synthesized pattern of the beamforming array using the Pyramidal horn for different values of \( \tau \) when \( h = 20 \) cm and \( h = 40 \) cm.
Figure 3.11: $E$-plane cuts of the synthesized pattern of the Pyramidal horn for different values of $\tau$ when (a) $h = 20$ cm and (b) $h = 40$ cm.
3.4 Synthetic Beamforming Array Pattern

Figure 3.12: 3 dB beamwidth and the sidelobe level of the E-plane cut versus τ: (a) open-ended waveguide and (b) Pyramidal horn.
Table 3.1: Parameters used for the pattern of the synthetic beamforming array.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Array</td>
<td>129 × 129</td>
</tr>
<tr>
<td>Antenna</td>
<td>5 cm × 5 cm Pyramidal horn</td>
</tr>
<tr>
<td>Spacing between antennas</td>
<td>0.25 λ</td>
</tr>
<tr>
<td>Frequency</td>
<td>8 GHz</td>
</tr>
<tr>
<td>Polarization</td>
<td>Linearly in y-direction</td>
</tr>
<tr>
<td>Standoff distance, $h$</td>
<td>20, 40 cm</td>
</tr>
<tr>
<td>Observation plane</td>
<td>1.2 m × 1.2 m</td>
</tr>
</tbody>
</table>

the output of the array (Eq. (3.8)) when the displacement is absent, and four different values of $\tau$ were used to adjust the width of the window function. In this example, it is assumed that the array beamforms the pattern at the center of the observation plane; the maximum output of the array is expected to occur at the center.

For both antennas (open-ended waveguide and Pyramidal horn), pseudo color graphs of the synthetic beamforming patterns are presented in Figs. 3.8 and 3.10, respectively. These graphs are plotted on a 100 dB scale. $E$-plane cuts are also shown as a function of $y$ with $x = 0$ in Figs. 3.9 and 3.11. As expected, it is seen that the maximum amplitude of the pattern is obtained at the center of the observation plane. The width of the main beam is seen to decrease with an increasing value of $\tau$ and decreasing $h$. This implies that a smaller beamwidth can be obtained when a broader window function is used and when the antenna is placed close to the observation plane. However, the sidelobe level behaves in an opposite manner. The sidelobe level increases with increasing $\tau$ and decreasing $h$.

In Fig. 3.12, the 3 dB beamwidth and the sidelobe level of the $E$-plane pattern (Figs. 3.9 and 3.11) for both antennas are plotted as a function of $\tau$ when $h = 20$ cm and $h = 40$ cm. Overall, as $\tau$ decreases, the sidelobe levels drop at the expense of wider beamwidths. Note that up to a certain value of $\tau$, the beamwidth rapidly decreases as $\tau$ increases. Beyond that point, the beamwidth converges to a constant. The sidelobe level behaves in a similar manner; the sidelobe level converges as $\tau$ increases.
Figure 3.13: A schematic diagram of the beamforming array with the surface displacement.

3.5 Reconstruction of Surface Displacements

To investigate the viability of the beamforming technique, the synthetic beamforming array was modeled with surface displacements. In this model, it simulates the signals received by the antenna due to the surface displacements, and the beamforming array was implemented to reconstruct displacements.

A schematic diagram of this model is shown in Fig. 3.13. All of the configurations are the same as Fig. 3.3 except for the surface displacement modeled as a circular bump with a diameter \( D = 20 \text{ cm} \) and height \( \Delta z = 1 \mu m \) on the observation plane. With parameters shown in Table 3.1, the surface displacement is calculated from the outputs of the array to demonstrate the effectiveness of the synthetic beamforming array. Graphs of the reconstructed displacement are shown in Fig. 3.14. The actual displacement and the displacements with and without beamforming are shown when the array is placed at 20 cm and 40 cm high. In these graphs, the displacement is plotted as a function of \( x \) with \( y = 0 \). Here, the displacement is obtained by comparing the phase between the data obtained with and without the displacement in the model.
The displacement obtained without beamforming is not a good replica of the actual displacement for either height. In order to solve this problem, the beamforming array technique was used to obtain a good image of the displacements. The raw data were beamformed for several values of \( \tau \). When the raw data are beamformed with the appropriate value of \( \tau \), the displacements are seen to be a better replica of the actual displacement than the results without beamforming. However, when the window function is very broad (\( \tau = 15.1 \lambda \) or \( \tau = 30.2 \lambda \)), the beamformed displacement is seen to have relatively large sidelobes and is not a good replica. This can be explained by observing that the spatial resolution is seen to increase with increasing \( \tau \), but the sidelobe levels also increase with increasing \( \tau \). Tradeoffs must be made between the spatial resolution and the sidelobe levels.

Pseudo color graphs of the amplitude of the displacement of the surface are presented in Fig. 3.15 for the entire observation plane. The displacements are shown for the data obtained with and without beamforming on a 40 dB scale. The sidelobes are clearly visible around the displacement in the raw data for both \( h = 20 \) cm and \( h = 40 \) cm. The sidelobe levels increase when the height of the array increases. The top of the displacement is seen to be non-uniform in the raw data for both heights. After beamforming with the appropriate \( \tau \), the sidelobe are seen to be significantly reduced, and the top of the displacement is seen to be more uniform.

3.6 Experimental Model

This section shows some results of a series of experiments to illustrate the feasibility of the synthetic beamforming array. These experiments include the measurement of the static and the dynamic surface displacements of the earth.

3.6.1 Measurement of Static Surface Displacements

An experimental model has been developed to measure static surface displacements of the ground. A schematic diagram of this experimental model is shown in Fig. 3.16.
Figure 3.14: Reconstruction of the displacement of the surface: (a) $h = 20$ cm and (b) $h = 40$ cm.
Figure 3.15: Pseudo color graphs of the surface displacements: (a) raw with $h = 20$ cm, (b) raw with $h = 40$ cm, (c) beamformed with $\tau = 3.8\lambda$ and $h = 20$ cm, and (d) beamformed with $\tau = 7.5\lambda$ and $h = 40$ cm.
3.6 Experimental Model

Figure 3.16: A schematic diagram of the measurement setup and grid of the region scanned.

It consists of a sandbox and a radar that is scanned over the surface of the sand to implement a synthetic beamforming array. The radar system used in this experiment is much simpler than the radar shown in Chapter 2. The radar uses an open-ended waveguide as the antenna, and a HP8720D network analyzer is used to measure the magnitude and phase of the ratio of the received to transmitted signal ($S_{21}$). The antenna was placed $5.33\lambda$ above the surface of the sand, and the radar was operated at a frequency of 8 GHz. The radar scans using an x-y positioner. The response of the radar is measured on a grid of $59 \times 59$ equally spaced points that are spaced $0.47\lambda$ apart. The scanned region is approximately 1 m square. In the sand below the scanned region, one large and two smaller symbols are dug into the sand. A photograph of the symbols is shown in Fig. 3.17(a). The depths of the large symbol and the smaller symbol are $0.16\lambda$.

The reconstructed height of the surface of the sand calculated from the raw measured data without beamforming is shown in Fig. 3.17(b). The symbols are seen to be very blurred. The reconstructed height of the surface of the sand calculated
Figure 3.17: Reconstructed surface of the sand when $h = 20$ cm: (a) photograph of the surface of the sand, (b) raw, (c) $\tau = 3.8\lambda$, (d) $\tau = 7.5\lambda$, (e) $\tau = 15.1\lambda$, and (f) $\tau = 30.2\lambda$.  

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Figure 3.18: A schematic diagram of the experimental model to measure the dynamic surface displacements.

using the beamforming is shown in Figs. 3.17(c) and (f) for four values of $\tau$. The results with $\tau = 3.8\lambda$, $7.5\lambda$ and $15.1\lambda$ are seen to be clearly better than the result without beamforming. Note that the resolution of the images increases with the increasing value of $\tau$ because of the decreasing beamwidth of the array. However, note that the noise in the image increases with increasing values of $\tau$ because of the increasing sidelobes of the array. The image with $\tau = 30.2\lambda$ is seen to be very noisy. This is predicted by the tradeoff between the resolution and the sidelobe level in Fig. 3.12 and the results in Fig. 3.14.

### 3.6.2 Measurement of Dynamic Surface Displacements

An experimental model has been developed to include extending the model to include time-varying displacements so that it can be incorporated into the mine detection system. The radar system shown in Fig. 2.33 was used in order to measure very small displacements of the surface of the sand and of the mine due to the elastic waves. In this experiment, the radar was used in the monostatic mode and used a 5 cm $\times$ 5 cm Pyramidal horn as the antenna. The radar is operated at 8 GHz and scans using an $x$-$y$ positioner. The radar is interfaced to a personal computer to control
the positioner and to store the measured data.

A schematic diagram of the experimental model is shown in Fig. 3.18. The model consists of a sandbox and a radar that is scanned over the surface of the sand to implement a synthetic beamforming array. The scanned area is 1.2 m square. A TS-50 anti-personnel mine is buried at the center of the area, and the top of the mine is 2.5 cm below of the surface of the sand. A transducer is placed on the surface of the sand and used to launch the elastic waves into the sand. These elastic waves travel across the surface of the sand and cause the surface of the sand and the mine to be displaced. The radar is used to measure the displacement of the surface of the sand due to the elastic waves. The response of the radar is measured on a uniform square grid of discrete positions in the scanned region. The grid consists of 65 × 65 equally spaced points that are spaced 1.875 cm apart. The output of the radar is measured as a function of time at each of these positions. The displacements of the surface of the sand are calculated from these measurements. Experiments have been performed for three different heights of the antenna. The antenna is placed at 5 cm, 20 cm, or 40 cm above the surface of the sand.

As for the experimental results, the surface displacements due to the elastic waves are plotted in Fig. 3.19 in a set of waterfall graphs. These results are raw measured data when the antenna is placed at $h = 5$ cm, $h = 20$ cm, and $h = 40$ cm above the surface of the sand. The mine is located at the center of the sandbox. In these graphs, the surface displacements are plotted as a function of time with $y = 0$. Each single plot is shifted vertically from the previous one. The region in which the mine is located is indicated by the gray shading. When $h = 5$ cm, quite strong oscillations are visible at the mine location. This result is a good representation of the actual displacements because the antenna is placed relatively close to the surface of the sand. When the height of the antenna increases, the amplitude of the oscillations decreases, and their shapes become blurred. Also, the surface displacements are seen to be very noisy.

These raw data are then beamformed with an appropriate value of $\tau$. Figure 3.20
Figure 3.19: Waterfall graphs of the raw measured surface displacements as a function of time. The antenna is placed at $h = 5$ cm, $h = 20$ cm, and $h = 40$ cm above the surface of the sand.

presents the waterfall graphs of the beamformed data. The raw data when $h = 20$ cm and when $h = 40$ cm are beamformed with $\tau = 3.8 \lambda$ and $\tau = 7.5 \lambda$, respectively. The value of $\tau$ was appropriately chosen so that the best image can be obtained; according to Fig. 3.12, when too small value of $\tau$ is selected, its resulting image might blur because of insufficient resolution. On the other hand, a noisy image may be expected with a big value of $\tau$ because of high sidelobe levels. It is seen that the beamformed results show a better replica of the actual surface displacements. These results are seen to be less noisy, and the shape of the oscillations at the mine location is seen to be clearly better than the raw data.

In order to better observe the effectiveness of beamforming, the pseudo color graphs of the amplitude of the displacement of the surface are shown in Fig. 3.21 for the entire scanned region at a single instant in time. These graphs are plotted on a 40
Figure 3.20: Waterfall graphs of the beamformed surface displacements with the appropriate value of $\tau$. The antenna is placed at $h = 5$ cm, $h = 20$ cm, and $h = 40$ cm above the surface of the sand.
3.6 Experimental Model

dB scale. These graphs show the displacements of the surface at a time at which the elastic wave had passed the mine. The displacements are shown for the raw data and for the beamformed data with $\tau = 3.8\lambda$ and $\tau = 7.5\lambda$ for $h = 20$ cm and $h = 40$ cm, respectively. Figures 3.21(a)-(c) show the raw measured data without beamforming for three different heights of the antenna. The elastic wave is traveling across the surface of the sand from left to right. The elastic wave has passed the mine buried at the center of the sandbox, and a portion of the wave is scattered from the mine. The displacements are biggest above the mine due to resonance within the mine. This resonance makes it much easier to detect the mine [36]. The mine is clearly visible at the center of the scanned region in Fig. 3.21(a). At this height, the raw data is a good representation of the actual displacement so that we can compare this result to the other cases. The spatial resolution becomes worse as the height of the radar is increased. As a consequence, the mine appears much bigger than its actual size, and some artifacts (sidelobes) are observed around the mine.

The synthetic beamforming array has been used to improve the spatial resolution for the raw measured data. The beamformed displacements are shown in Fig. 3.21(d) and (e) when $h = 20$ cm and $h = 40$ cm. The raw data was beamformed with $\tau = 3.8\lambda$ for $h = 20$ cm and with $\tau = 7.5\lambda$ for $h = 40$ cm. In both Fig. 3.21(d) and (e), the results are seen to be clearly better than the results without beamforming. Since the resolution has been improved, the location and the size of the mine are more evident in these graphs. Moreover, after beamforming, the artifacts around the mine have been significantly reduced.
Figure 3.21: Pseudo color graphs of the displacement of the surface: (a) raw when $h = 5$ cm, (b) raw when $h = 20$ cm, (c) raw when $h = 40$ cm, (d) beamformed with $\tau = 3.8\lambda$ when $h = 20$ cm, (e) beamformed with $\tau = 7.5\lambda$ when $h = 40$ cm.
CHAPTER 4

Physical Array of N Antennas

4.1 Introduction

The synthetic array configuration described earlier has a disadvantage: The synthetic array using a single antenna needs a large amount of scanning time to record the returned signal over the entire scanned region. In order to improve the scanning speed, a physical array of simple antennas has been studied. The advantage of this array arises from the fact that an N-antenna physical array reduces by a factor of N the number of measurements needed to scan the target area.

Using a simple antenna such as an isotropic or an aperture antenna, two physical array configurations have been investigated by the use of theoretical models. The physical array configurations under consideration are divided into two main categories of one-way focus and two-way focus. In one-way focus, the array is focused at an arbitrary focal point only during the receiving mode; in two-way focus, the array is focused at an arbitrary focal point in both transmitting and receiving modes.

Figure 4.1 is a schematic diagram of the N-antenna one-way focusing array. The antennas are placed at height $h$ above the surface of the earth and are assumed to be identical. The surface of the earth is assumed to be illuminated by a plane wave. The array then receives the reflected signal from the surface of the earth. Then, the received signal at each antenna is added with an appropriate weighting function to focus the array at an arbitrary focal point in the receiving mode. This one-way focusing array can help to improve the spatial resolution at a standoff distance for the radar. Also, the scanning speed is reduced by a factor of N. A focusing algorithm
Figure 4.1: A one-way focusing array configuration.

Figure 4.2: A two-way focusing array configuration.
can be implemented with the array in both the software and hardware.

Figure 4.2 shows a configuration of the N-antenna two-way focusing array. Unlike the one-way focusing array, the array excites the electromagnetic waves and receives the signal reflected off the surface of the earth. Here, an appropriate weighting function is given to each antenna in the array so that the array is focused at an arbitrary focal point in both the transmitting and receiving modes. In this array, the scanning time can be reduced only by implementing a hardware beam former, which can be expensive and complex.

In this research, theoretical models have been developed for both one-way and two-way focusing arrays. The theoretical models are based on the integral equation method. For both one-way and two-way focusing arrays, appropriate weighting functions are chosen to focus the arrays at an arbitrary focal point, and the radiation pattern and the received pattern of the array are computed to examine the spatial resolution (beamwidth) of the array.

4.2 One-Way Focus: Plane Wave Excitation

4.2.1 Theoretical Model and One-Way Focusing Algorithm

Figure 4.3 is a schematic diagram of an N-antenna one-way focusing array. The antennas are placed at height \( h \) above the \( x-y \) plane and on a plane parallel to the \( x-y \) plane, assumed to be identical, and located at the positions \( R_i \). The surface of the earth is modeled as a planar electric conducting surface. The conducting surface is assumed to be illuminated by a normally incident plane wave. Due to the uniform plane wave excitation, a constant surface current density is obtained over the entire conducting surface. Then, the constant surface current can be replaced with a point source. For the isotropic source located at the position \( \tilde{R} \), the voltage received by the \( i^{th} \) antenna can be expressed as

\[
V_i(\tilde{R}) = C \frac{\cos \psi_i}{|\tilde{R} - \tilde{R}_i|} e^{-jk|\tilde{R} - \tilde{R}_i|},
\]  

(4.1)
4.2 One-Way Focus: Plane Wave Excitation

![Diagram of a one-way focusing array]

Figure 4.3: A theoretical model of one-way focusing array.

where $C$ is a constant. Here, the electric field is treated as a scalar, and the antennas are assumed to have the simple angular dependence $\cos \psi_i$, where $\psi_i$ is the angle between the propagation direction and the $z$-direction. The terminal voltages of the antennas of the array are summed with the appropriate weighting function to focus the array at the position $\vec{R}_f$. The phase of the weighting function is chosen so that the voltages will add constructively when the source is at the position $\vec{R}_f$, and the amplitude of the weighting function is chosen to control the sidelobe levels of the array. The output of the array is then

$$V_A(\vec{R}, \vec{R}_f) = \sum_{i=1}^{N} V_i(\vec{R}) W(\vec{R}_i, \vec{R}_f) e^{jk|\vec{R}_f - \vec{R}_i|},$$  \hspace{1cm} (4.2)

where $W$ is a window function used to adjust the amplitude of the weighting function. We have determined empirically that a Gaussian window function performs well:

$$W(\vec{R}_i, \vec{R}_f) = \exp \left( - \frac{1}{(0.6\tau)^2} ||\vec{z} \times (\vec{R}_f - \vec{R}_i)||^2 \right),$$  \hspace{1cm} (4.3)

where $\tau$ is a parameter used to adjust the width of the window function at the 50 percent drop point from the peak. The window function is at maximum directly above $\vec{R}_f$, $||\vec{z} \times (\vec{R}_f - \vec{R}_i)|| = 0$, and it decreases with increasing distances from this point.
4.2.2 Numerical Results

As an example, a planar array is chosen to consist of 65 by 65 identical antennas equally spaced on a square grid; they are spaced 0.5\(\lambda\) apart. The array is centered on the point \(x = 0\) and \(y = 0\). The array is placed at height \(h = 5.33\lambda = 20\) cm. The isotropic source is scanned from \(y = -14\lambda = -52.5\) cm to \(y = 14\lambda = 52.5\) cm with \(x = 0\) and \(z = 0\). The array is focused at \(x = y = z = 0\).

A graph of the output of the array as a function of the source position for several values of \(\tau\) is shown in Fig. 4.4. Since the array is focused at the center of the scan, the maximum output is obtained when the isotropic source is located at the center of the scan as expected. The sidelobe levels are seen to be quite high, and the beamwidth is seen to be very narrow when the window is very broad (\(\tau = 30.2\lambda = 113.3\) cm). As \(\tau\) decreases, the sidelobe levels drop at the expense of wider beamwidths.

Figure 4.5 shows the output of the array with \(\tau = 7.5\lambda = 28.13\) cm as a function of the \(y\)- and \(z\)-coordinates of the isotropic source. The configuration of the array is the same as the above 3-D case with \(h = 5.33\lambda = 20\) cm. The array is seen to be focused at the origin; the beamwidth is narrowest along the line \(z = 0\). However, note that the beamwidth does not change significantly when \(z\) is varied in a range of approximately \(-\lambda < z < \lambda\).

The focal point is scanned over the surface of the area of interest by adjusting the beamforming parameters. This procedure can be performed in a software manner. This allows the scanning time required by the radar to be reduced by a factor of the number of antennas populating the array. Figure 4.6 shows the output of the array when the array is focused at different locations. The array configuration is the same as described earlier, and the array is placed at \(h = 5.33\lambda\) high, and \(\tau = 7.5\lambda\) for the window function. The graphs are plotted for four different coordinates of the focal points: \(x_f = 0, y_f = -10.67\lambda, y_f = -5.33\lambda, y_f = 0, y_f = 5.33\lambda\), and \(y_f = 10.67\lambda\). It is seen that the maximum output of the array occurs at each shifted focal point. The pattern is perfectly symmetrical when the focal point is placed at the origin.
4.2 One-Way Focus: Plane Wave Excitation

Figure 4.4: The output of the array as a function of the source position for different values of $\tau$ when $h = 5.33\lambda = 20$ cm.

Figure 4.5: The output of the array when $h = 5.33\lambda = 20$ cm and $\tau = 7.5\lambda = 28.13$ cm.
4.2 One-Way Focus: Plane Wave Excitation

Figure 4.6: The output of the array with different locations of the focal point when $h = 5.33\lambda = 20$ cm and $\tau = 7.5\lambda = 28.13$ cm.

However, note that the pattern is seen to be slightly asymmetric as the offset point is moved away from the origin.

4.2.3 Tradeoff between Beamwidth and Sidelobe Level

In this section, a tradeoff between the first sidelobe and the beamwidth is investigated for the one-way focusing array. The 3 dB beamwidth and the first sidelobe level were obtained from the received pattern. In Fig. 4.8, the 3 dB beamwidth and the first sidelobe level are plotted as a function of $\tau$ for three different values of $h$. The beamwidth is seen to decrease when $\tau$ is increased and when $h$ is decreased. However, the first sidelobe level behaves in the reverse; the first sidelobe level is seen to increase when $\tau$ is increased and when $h$ is decreased. Figure 4.7 is a similar graph for a 2-D case. Here, the antennas and the source are assumed to have an infinite length in the $x$-direction. In this configuration, the array consists of 65 identical antennas along the $y$-direction only. The results for the 2-D case are very close to those for the 3-D
4.2 One-Way Focus: Plane Wave Excitation

Figure 4.7: The beamwidth and the first sidelobe level versus $\tau$: 2-D case.

Figure 4.8: The beamwidth and the first sidelobe level versus $\tau$: 3-D case.
4.3 Two-Way Focus

The algorithm of the two-way focusing array is similar to the method in the one-way focusing array. The major difference is that all array antennas act not only as transmitters but also as receivers, and the focusing procedure is performed in both transmitting and receiving modes. The configuration of the two-way focusing array was shown in Fig. 4.2.

4.3.1 Theoretical Model and Two-Way Focusing Algorithm

A schematic diagram and its coordinate system of the array are shown in Fig. 4.9. The array consists of $I \times J$ antennas that are spaced $d$ apart. The array is placed at height $h$ above an observation plane and parallel to the $x$-$y$ axis. The antennas are located at the positions $\vec{R}_i$. All antennas are assumed to be identical. The observation plane is assumed to be a plane that includes a small conducting region. This conducting region is scanned over a grid that is equally spaced on the plane. The array is focused at an arbitrary focal point at the position $\vec{R}_f$.

Let us assume that electric and magnetic current densities $\vec{J}_i$ and $\vec{M}_i$ on the aperture of each antenna are known. Then, using Eqs. (3.1)-(3.3), the electric field radiated by each antenna, ignoring the mutual coupling among neighboring antennas, can be computed at the position $\vec{R}$ at the position of the small conducting region on the observation plane. The electric field by each antenna is summed with the appropriate weighting function to focus the array at the focal point in the transmitting mode. The phase of the weighting function is chosen so that the electric fields will add constructively by using the conjugate focusing method, and the amplitude of the weighting function is chosen to control the sidelobe levels of the radiated field. The total electric field at the position $\vec{R}$ radiated by the array is then

$$
\vec{E}_{\text{tot}}(\vec{R}) = \sum_{i=1}^{I} \sum_{j=1}^{J} \vec{E}_{ij}(\vec{R}) W(\vec{R}_f, \vec{R}_f') e^{j\alpha_{ij}(\vec{R}_f)},
$$

(4.4)
Figure 4.9: A theoretical model of the two-way focusing array.

where $\vec{E}_{ij}(\vec{R})$ is the electric field at the position $\vec{R}$ radiated by the $i^{th}$ and $j^{th}$ antenna, $W$ is a window function used to adjust the amplitude of the weighting function, and $\alpha_{ij}(\vec{R}_f) = (-1)\{\text{phase of } \vec{E}_{ij}(\vec{R}_f)\}$. The function $W$ is exactly the same as Eq. (3.9). Assuming the observation plane is in the far-field of the source, the total magnetic field is then

$$\vec{H}_{tot}(\vec{R}) = \frac{1}{\eta_o} \sum_{i=1}^{I} \sum_{j=1}^{J} \hat{s}_{ij} \times \vec{E}_{ij}(\vec{R}) W(\vec{R}_f, \vec{R'}) e^{j\alpha_{ij}(\vec{R}_f)},$$

(4.5)

where $\hat{s}_{ij} = \frac{\vec{R}}{|\vec{R}|}$ (radial unit vector of the $i^{th}$ and $j^{th}$ antenna). Next, using the equivalence principle, the total surface current density on the small conducting region is obtained from the tangential components of the electric and magnetic fields on the surface of the conducting region. Since the surface of the conducting region is approximately planar, the surface current densities are approximated by

$$\vec{J}^{eq}_{tot}(\vec{R}) = -\hat{z} \times \vec{H}_{tot}(\vec{R}),$$

$$\vec{M}^{eq}_{tot}(\vec{R}) = \hat{z} \times \vec{E}_{tot}(\vec{R}).$$

(4.6)

Let $\vec{E}_{ij}(\vec{R})$ be the electric field at the center of the $i^{th}$ and $j^{th}$ antenna due to these
current elements at the location $\vec{R}$. $\vec{E}_{ij}^r(\vec{R})$ can be obtained from Eqs. (3.1) - (3.3) with Eq. (4.6). Then, the voltage received by the the $i^{th}$ and $j^{th}$ antenna is given by

$$V_{ij}(\vec{R}) = \vec{E}_{ij}^r(\vec{R}) \cdot \vec{r}_{ij}(\vec{R}),$$

(4.7)

where $\vec{r}_{ij}(\vec{R})$ is the vector effective height (VEH). Here, the VEH is used to determine the voltage induced on the open-circuit terminals of the antenna from the received field. The VEH is determined from the radiated field at the position $\vec{R}$, $\vec{E}_{ij}(\vec{R})$ [33]:

$$\vec{r}_{ij}^e(\vec{R}) = \alpha \frac{4\pi |\vec{r}| \vec{E}_{ij}(\vec{R})}{-j\eta_k \epsilon_0 e^{-j\beta_0 |\vec{r}|}},$$

(4.8)

where $|\vec{r}|$ is a distance between the $i^{th}$ and $j^{th}$ antenna and the current element at the position $\vec{R}$. In order to focus the array at the position $\vec{R}_f$ in the receiving mode as well, another weighting function must be applied. The output of the array is then

$$V_A(\vec{R}) = \sum_{i=1}^{I} \sum_{j=1}^{J} V_{ij}(\vec{R}) W(\vec{R}_f, \vec{R}) e^{j\beta_{ij}(\vec{R}_f)},$$

(4.9)

where $W(\vec{R}_f, \vec{R})$ is the same as Eq. (3.9), and $\beta_{ij}(\vec{R}_f) = (-1)\{\text{phase of } V_{ij}(\vec{R}_f)\}$.  

### 4.3.2 Numerical Results

Parameters used in the example of the theoretical model are shown in Table 4.1. Using the procedure described above, the two-way patterns ($|V_A(\vec{R})|^2$) of the array are computed. The array is focused at $x = y = 0$ and $z = h$. The two-way patterns of the array are shown in Figs. 4.10 and 4.11 when $h = 20$ cm and $h = 40$ cm, respectively. In these graphs, the patterns are plotted as a function of $x$ with $y = 0$ for different values of $\tau$. The sidelobe levels are seen to be quite low, and the beamwidth is seen to be very narrow when the window function is very broad ($\tau = 30.2\lambda$). As $\tau$ decreases, the sidelobe levels drop at the expense of wider beamwidths. When $h$ increases, the beamwidth is seen to increase, and the sidelobe levels are seen to decrease slightly.

Pseudo color graphs of the patterns are presented in Figs. 4.13 and 4.12 for the entire observation plane. In these graphs, the patterns are plotted as a function of
4.3 Two-Way Focus

Table 4.1: Parameters used in the example of the two-way focusing real array.

<table>
<thead>
<tr>
<th>Array</th>
<th>$21 \times 21$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Antenna</td>
<td>1 cm $\times$ 2.3 cm rectangular waveguide</td>
</tr>
<tr>
<td>Polarization</td>
<td>Linear pol. in $x$-direction</td>
</tr>
<tr>
<td>Spacing between antennas</td>
<td>2.5 cm in $x$-direction and 2.49 cm in $y$-direction</td>
</tr>
<tr>
<td>Frequency</td>
<td>8 GHz</td>
</tr>
<tr>
<td>Array height, $h$</td>
<td>20 cm and 40 cm</td>
</tr>
<tr>
<td>Observation plane</td>
<td>1.2 m $\times$ 1.2 m</td>
</tr>
</tbody>
</table>

$x$ and $y$ when $\tau = 15.1\lambda$. The main beams are very clearly visible at the center of the plane. The main beam is seen to increase in size when the height of the array increases.

In Fig. 4.14, the 3 dB beamwidth is plotted as a function of $\tau$ when the array is placed at $h = 20$ cm and $h = 40$ cm. The beamwidths are shown for the patterns Figs. 4.10 and 4.11. The beamwidth is seen to decrease with increasing $\tau$ and $h$. These graphs show sufficiently narrow beamwidths when $\tau$ is greater than $7.5\lambda$. 

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4.3 Two-Way Focus

Figure 4.10: A graph of the two-way pattern for different \( \tau \) when \( h = 20 \text{ cm} \).

Figure 4.11: A graph of the two-way pattern for different \( \tau \) when \( h = 40 \text{ cm} \).
Figure 4.12: A pseudo color graph of the two-way pattern with $\tau = 15.1\lambda$ when $h = 20\text{ cm}$.

Figure 4.13: A pseudo color graph of the two-way pattern with $\tau = 15.1\lambda$ when $h = 40\text{ cm}$.
Figure 4.14: A graph of the 3 dB beamwidth as a function of $\tau$ when $h = 20$ cm and $h = 40$ cm.
CHAPTER 5
FDTD Method

5.1 Introduction

A two-dimensional finite-difference time-domain (FDTD) model has been developed to examine the feasibility of detecting surface displacements. A diagram of the model is shown in Fig. 5.1. A total field/scattered field formulation was used to inject an electromagnetic plane wave, a uniaxial perfectly matched layer [37] was used to truncate the boundaries, and the earth was modeled as a lossy half space. The surface of the earth is displaced because of an elastic wave traveling through the earth. These displacements are included in the model. The electromagnetic plane wave is reflected from the boundary between the air and the earth, and the total field (both incident and reflected waves) is recorded at the observation points. Then, the surface displacements are reconstructed from the reflected waves.

This chapter describes detailed information about the finite-difference time-domain (FDTD) method and about its application to the model for detecting surface displacements.

5.2 2-D FDTD Update Equations with Lossy Medium

The FDTD method has been widely used to model wave propagations, scattering, antennas, high-speed circuits, etc. The FDTD method was first developed by Yee in 1966 [38]. This method is a way to solve Maxwell’s equations for electric and magnetic fields in the time domain. Much detailed information for the FDTD method can be
Figure 5.1: Geometry for FDTD model being investigated.

found in [39] and [40].

In the FDTD method, each component of the electric and magnetic fields is discretized. For the two-dimensional FDTD method with the transverse magnetic (TM) wave case, only three field components are needed: $H_z$, $E_x$, and $E_y$. The Yee cell for this case is shown in Fig. 5.2. Each magnetic field is surrounded by four electric field components. The field components are placed in offset locations, apart by $\Delta x$ and $\Delta y$, in the $x$- and $y$-directions, respectively. Figure 5.3 shows indices for the position of each field component within the Yee cell. The indices $i$ and $j$ are introduced for the position of each component in the $x$- and $y$-directions, respectively. Using these indices, all positions of each component are uniquely defined in the FDTD grid. The index $n$ indicates the discrete time of each component. In other words, each field component is a function of position, indicated by $i$ and $j$, and time $n$. At the time steps of $n\Delta t$, the electric field components are determined, and the magnetic field components are determined at times $(n+1/2)\Delta t$. The electric and magnetic field components are updated in a “leapfrog” manner [40]; magnetic field components at times $(n+1/2)\Delta t$ are obtained from a combination of magnetic
5.2 2-D FDTD Update Equations with Lossy Medium

Figure 5.2: A Yee cell for the two-dimensional FDTD method.

fields at times \((n - 1/2)\Delta t\) and from four surrounding electric field components at times \(n\Delta t\). Similarly, electric field components at time steps of \((n+1)\Delta t\) are updated from a combination of electric fields at time steps of \(n\Delta t\) and adjunct magnetic field components.

The update equations for the two-dimensional FDTD method with the TM set can be obtained from Faraday’s and Ampere’s laws. Faraday’s law states that

\[
\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}, \quad (5.1)
\]

where \(\mu = \mu_0\mu_r\) is the permittivity of a medium. For the TM case, only \(H_z, E_x,\) and \(E_y\) components are of interest. Thus, Eq. (5.1) can be written as

\[
\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\mu \frac{\partial H_z}{\partial t}. \quad (5.2)
\]

Ampere’s law is written as

\[
\nabla \times \vec{H} = \epsilon \frac{\partial \vec{E}}{\partial t} + \sigma \vec{E}, \quad (5.3)
\]

where \(\epsilon = \epsilon_0 \epsilon_r\) and \(\sigma\) is the permittivity and the conductivity of a medium, respectively. Again, only the \(H_z, E_x,\) and \(E_y\) components are of interest. Therefore,
Figure 5.3: Indices of the field components for the two-dimensional FDTD cell.

Eq. (5.3) becomes

\[
\frac{\partial H_z}{\partial y} = \frac{\epsilon}{\Delta t} \frac{\partial E_x}{\partial t} + \sigma E_x,
\]

\[
\frac{\partial H_z}{\partial x} = -\frac{\epsilon}{\Delta t} \frac{\partial E_y}{\partial t} - \sigma E_y. \tag{5.4}
\]

To obtain the FDTD update equations, the derivatives in the above equations are discretized in temporal and spatial spaces by using the central-difference approximations. For example, the spatial derivative of \( E_x \) can be approximated by

\[
\frac{\partial E_x}{\partial x} \approx \frac{E^n_x(i, j + 1/2) - E^n_x(i, j - 1/2)}{\Delta x}, \tag{5.5}
\]

and for the temporal derivative of \( H_z \),

\[
\frac{\partial H_z}{\partial t} \approx \frac{H^{n+1/2}_z(i, j) - H^{n-1/2}_z(i, j)}{\Delta t}. \tag{5.6}
\]

The notation of the update equations, for example, is expressed as \( H_z(x, y, t) = H_z(i \Delta x, j \Delta y, n \Delta t) = H^n_z(i, j) \). Finally, using the above approximations with Eqs. (5.2) and (5.4), the standard FDTD update equations are derived:

\[
H^{n+1/2}_z(i, j) = H^{n-1/2}_z(i, j) - \frac{\Delta t}{\mu \Delta x} \left[ E^n_y(i + 1/2, j) - E^n_y(i - 1/2, j) \right]
\]
\[ E_x^{n+1}(i, j + 1/2) = \frac{\epsilon - \frac{1}{2}\sigma \Delta t}{\epsilon + \frac{1}{2}\sigma \Delta t} E_x^n(i, j + 1/2) \]
\[ + \frac{\Delta t}{\Delta y(\epsilon + \frac{1}{2}\sigma \Delta t)} \left[ H_z^{n+1/2}(i, j + 1) - H_z^{n+1/2}(i, j) \right] \]  \hspace{1cm} (5.7)

\[ E_y^{n+1}(i + 1/2, j) = \frac{\epsilon - \frac{1}{2}\sigma \Delta t}{\epsilon + \frac{1}{2}\sigma \Delta t} E_y^n(i + 1/2, j) \]
\[ - \frac{\Delta t}{\Delta x(\epsilon + \frac{1}{2}\sigma \Delta t)} \left[ H_z^{n+1/2}(i + 1, j) - H_z^{n+1/2}(i, j) \right] \] \hspace{1cm} (5.8)

Next, the boundary between two different media—for example, the interface between the air and the earth—is considered. For the TM set, tangential field components \( E_x \) and \( E_y \), in general, are placed on the boundary. When \( E_x \) and \( E_y \) components are updated on the boundary, \( \epsilon_r \) and \( \sigma \) in Eqs. (5.8) and (5.9) must be replaced with new values, for example, an average value of those in two media. However, Eq. (5.7) is not modified because \( H_z \) is not placed on the boundary.

To guarantee numerical stability in the FDTD lattices, the time step \( \Delta t \) must meet the Courant condition:

\[ \Delta t \leq \frac{1}{v \sqrt{\frac{1}{(\Delta x)^2} + \frac{1}{(\Delta y)^2}}} , \] \hspace{1cm} (5.10)

where \( v \) is the velocity in the medium. If \( \Delta x = \Delta y = \Delta \), the condition simplifies to

\[ \Delta t \leq \frac{\Delta}{v\sqrt{2}} \] \hspace{1cm} (5.11)

In the FDTD method, it is known that there exists a numerical dispersion [40]; therefore, when a plane wave propagates through a FDTD lattice, it propagates at the wrong speed. It propagates at the numerical velocity \( \hat{v} \) instead of the physical velocity \( v \). For a time harmonic plane wave, the numerical velocity is a function of the node density (\( \frac{\Delta}{\Delta x} \) and \( \frac{\Delta}{\Delta y} \)) and the direction of propagation.
5.2 2-D FDTD Update Equations with Lossy Medium

![Diagram showing incident plane wave, scatterer, back-scattered wave, and zoned regions](image)

Figure 5.4: Zoning of the total-field/scattered-field formulation.

5.2.1 Total-Field/Scattered-Field Formulation

The total-field/scattered-field formulation was first proposed as a way to create a compact wave source for the FDTD method [41][42]. This formulation is very effective to generate a plane wave source. Furthermore, this formulation helps to accurately treat back-scattered signals in a FDTD model. The fundamental concept of this formulation is shown in Fig. 5.4. The region of interest is zoned by a total-field region and a scattered-field region. A plane wave is launched at the total-field/scattered-field interface. The plane wave propagates through the FDTD grids and is scattered back from a scatterer. During this procedure, both the incident plane wave and the back-scattered wave exist in the total-field region. However, in the scattered-field region, only back-scattered waves exist. This feature allows the enhancement of the computational flexibility and dynamic range of the FDTD method. The total-field/scattered-field formulation can be realized by modifying the standard FDTD update equations. This modification is needed only when the equations are updated at the total-field/scattered-field interface. A detailed description of this formulation can be found in [40].

In this model, a plane wave was excited at the total-field/scattered-field interface.
Figure 5.5: Differentiated Gaussian pulse with a center frequency of 8 GHz and its spectrum.

The shape of the plane wave is a differentiated Gaussian pulse, which is a function of the time step $n\Delta t$:

$$g(n\Delta t) = \frac{t_0}{e^{-0.5}} \left[ -\frac{1}{t_0^2} (n\Delta t - t_d) e^{-0.5\left[\frac{n\Delta t - t_d}{t_0}\right]^2} \right], \quad (5.12)$$

where $t_d$ and $t_0$ are the time delay and the characteristic time of the pulse, respectively. The value of $t_0$ is dependent on the center frequency, $\omega_c$, of the pulse: $t_0 = \frac{1}{\omega_c}$. Figure 5.5 shows a differentiated Gaussian pulse with the center frequency of 8 GHz and its spectrum in the frequency domain. Here, $t_d$ was chosen as $t_d = 5t_0$.

### 5.2.2 Perfectly Matched Layer

In this model, a perfectly matched layer (PML) has been used as an absorbing boundary that truncates the FDTD lattices. The PML boundary condition was introduced by Berenger [43] and has been widely used in the FDTD method. The PML developed by Berenger was achieved by splitting field components, which is quite complicated. In 1996, Gedney [37] developed a PML that is based on a lossy uniaxial medium. This method is equivalent to Berenger’s, but it is in a simpler form because it avoids field splitting. The reflection error from the PML is dependent on PML parameters such
as the number of the FDTD cells in the PML. This PML boundary condition provides a reflection error in the range more than $-100$ dB depending on the parameters chosen. In this work, Gedney’s method was used.

Figure 5.6 depicts a schematic drawing of the PML in the FDTD lattices with the TM set. A PML consists of a lossy material in which propagating waves are attenuated within the PML. The PML is truncated by a perfect electric conductor (PEC) at the end of the PML. Within the PML, the degree of attenuation of the propagating field increases as the waves go deeper into the layer. This can be made possible by choosing a spatially varying conductivity, as described in [43], and by modifying the FDTD update equations in the PML. These modifications are dependent on the location of the PML with respect to each field component. Full formulations for these modifications are given in Appendix B.

The conductivity within the PML is chosen as a function of the position:

$$\sigma(x) = \frac{\sigma_{\text{max}} |x - x_0|^m}{d^m}, \quad (5.13)$$

where $x_0$ is the $x$-coordinate of the interface between the PML and a normal medium, $d$ is the total depth of the PML, and $m$ is order of the polynomial variation; $d$ is dependent on the number of the Yee cells in the PML; $m$ is chosen as 4 for the minimum reflection error, and $\sigma_{\text{max}}$ is given as $\sigma_{\text{max}} = \frac{m+1}{15\pi rd \sqrt{\varepsilon_r}}$ [44]. A large enough number of Yee cells must be chosen so that the PML is able to minimize the reflection error. In general, the number of Yee cells must be more than 10. The spatially varying conductivity is also computed at the location of each field component using Eq. (5.13) with 5 cells as shown in Fig. 5.6. At the interface between the normal medium and the PML, $\sigma(x)$ is equivalent to the conductivity of the normal medium, $\sigma_{\text{normal}}$; $\sigma(x)$ increases exponentially as $x$ increases, and it has a maximum value at the last lattice of the PML.
Figure 5.6: A schematic diagram of the PML and the spatial varying conductivity within the layer.
5.3 FDTD Model for Surface Displacements

Using the FDTD method described above, a two-dimensional finite-difference time-domain (FDTD) model has been developed to examine the feasibility of detecting surface displacements. This model was shown in Fig. 5.1. The earth was modeled as a lossy half space. The relative dielectric constant and the conductivity of the earth were chosen as $\varepsilon_r = 8.0$ and $\sigma = 0.8 \text{ s/m}$, respectively. The FDTD cell size in the air and the earth is $\Delta x = \Delta y = 6.622 \times 10^{-4} \text{ m}$, and $\Delta t = 1.104 \times 10^{-12} \text{ seconds}$. The total number of nodes for the FDTD grids are 630 in the $x$-direction by 1,518 in the $y$-direction, which yields a computational region dimension of approximately 41.72 cm $\times$ 100.52 cm. The thickness of the earth was designated as thin as possible so that the observation can be made as far as possible from the surface of the earth. In this model, the thickness of the earth is approximately 1.16 cm. A plane wave is injected at the total-field/scattered-field interface. The plane wave is given as a differentiated Gaussian pulse with a center frequency of 8 GHz. A large enough number of time steps was chosen to guarantee the round trip time for the incident wave.

The surface displacements at the interface between two different media (e.g., air-earth interface) would seem to be very difficult to model in the FDTD method because they are very small compared to the size of a FDTD cell. However, it has been shown that displacements as small as $10^{-14}$ times smaller than a FDTD cell can be modeled accurately using the standard averaging scheme [45]. The actual displacements due to the elastic wave are many orders of magnitude larger than this and can be easily modeled. Also, the FDTD update equations must be modified to model surface displacements; a standard average is taken to compute $\varepsilon_r$ and $\sigma$ for the displacements. Figure 5.7 shows two displacement arrangements in the FDTD grids. As shown in Fig. 5.7, displacement $A$ refers to the boundary of the displacement which is placed $\delta$ apart in the $-x$-direction from the $E_y$ at the $i^{th}$ column, while displacement $B$ refers to the reverse. For both cases, the relative permittivity to be
used in update equations can be written as

\[ \epsilon_r = \frac{\left( \frac{\Delta x}{2} - \delta \right) \epsilon_1 + \left( \frac{\Delta x}{2} + \delta \right) \epsilon_2}{\Delta x}. \] (5.14)

For displacement \( A, \delta > 0, \) and for displacement \( B, \delta < 0. \) The same method is used for conductivity.

The surface displacements are modeled as a differentiated Gaussian pulse with the maximum amplitude of 1 \( \mu m \) and a center frequency of 800 Hz, and its velocity is 80 m/sec. The same expression as Eq. (5.12) was used as a function of position. As an example, this surface displacement is shown in Fig. 5.8. It is placed along the \( y \)-direction and has the maximum amplitude of 1 \( \mu m \) and a peak to peak distance of approximately 6 cm.

### 5.4 Detecting Surface Displacements

The FDTD model for detecting surface displacements was shown in Fig. 5.1. The earth was modeled as a lossy half space. The surface of the earth is displaced due to an elastic wave traveling in the earth. The elastic wave travels much slower than the electromagnetic wave. Thus, because of the number of time steps that would be required, it is impractical to run the FDTD simulation for the time it would take the elastic wave to travel across the model. Fortunately, this is not necessary. In a given period of time, the elastic wave travels \( 10^{-6} \) times as far as the electromagnetic wave. The elastic wave will also travel a very short distance with respect to its width during the total travel time of the electromagnetic wave. Thus, it is possible to decouple the time scales for the elastic and the electromagnetic waves. The electromagnetic simulation is run with the elastic wave at discrete positions on the boundary. Surface displacements due to the elastic wave were modeled as a differentiated Gaussian pulse in the FDTD model.

The electromagnetic plane wave in the shape of a differentiated Gaussian pulse is injected at the upper interface of total/scattered fields interfaces. The incident
Figure 5.7: Displacement arrangements in the FDTD grids.
Figure 5.8: Surface displacement with the shape of a differentiated Gaussian pulse. pulse was shown in Fig. 5.5. This plane wave propagates through the computational region in the $x$-direction toward the earth. The plane wave is reflected back from the boundary between the air and the earth, and part of the plane wave keeps propagating into the earth. Since the earth is a lossy medium, the amplitude of the propagating wave decreases as it propagates deeper within the earth. Both waves, reflected and transmitted, are then totally absorbed by the PML surrounding the computational region. Figure 5.9 shows snapshots of the $E$-field distributions in the total computational region at four instants in time and shows their cross sections as a function of $x$. At $T_1$ and $T_2$, the plane wave propagates toward the boundary, and it has not yet reached the boundary. At $T_3$, the graph shows the both reflected and transmitted pulses. The transmitted pulse is seen to be decreased because of the lossy medium, and it is about to be absorbed by the PML. At $T_4$, the graph shows only the reflected wave from the boundary. The amplitude is seen to be decreased due to the associated reflection coefficient. Figure 5.10 shows the total fields (incident and reflected pulses). The graph was plotted as a function of time when an observation point is located at height $h = 20$ cm above the boundary. It is seen that after the incident wave reached
Figure 5.9: Pseudo color graphs of the incident and reflected waves at four instants in time. The waves are plotted for the total computational region. At the bottom of each snapshot, a cross section is plotted as a function of $x$. 
Figure 5.10: Incident and reflected pulses as a function of time at the observation point placed at 20 cm above the boundary.

the boundary, it is reflected in inverted form. Since a large enough distance between the observation points and the boundary was chosen, the reflected pulse is not seen as overlapping the incident pulse.

Now, only the reflected wave is of interest because it includes all of the information about the displacements. The reflected pulse is recorded at the observation points both with and without the surface displacements. For both cases, the pure reflected wave is obtained by subtracting the incident wave from the total field (incident and reflected waves). The phase of the reflected signal is then obtained by Fast Fourier Transform (FFT). Next, one can obtain the phase difference between two sets of data obtained with and without the displacements by subtracting the phase with displacements from the phase without displacements. The surface displacements are then reconstructed from this phase difference. The brief procedure for reconstructing surface displacements is illustrated in Fig. 5.11.
5.4 Detecting Surface Displacements

Run FDTD and record reflected signal without displacements

FFT

\[ \Delta \varphi = \varphi \text{ (without displacements)} - \varphi \text{ (with displacements)} \]

\[ \Delta x = \Delta \varphi \lambda / \pi \]

Reconstructed surface displacements

Figure 5.11: Procedure to reconstruct surface displacements.
Figure 5.12: A schematic diagram of the unfocused antenna simulated in the FDTD model.

Figure 5.13: Actual and computed boundary displacements using the unfocused antenna when \( a = 2h \).
5.4 Detecting Surface Displacements

5.4.1 Simulation of Scanning Unfocused Antenna

As for the first example, a scanning unfocused antenna is simulated in the FDTD model to detect surface displacements. A schematic diagram of this antenna is shown in Fig. 5.12. In the FDTD model, a finite number of observation points is arrayed to simulate this antenna. This antenna is placed at height $h$ above the surface of the earth, and the dimension of the chosen antenna is two times greater than height $h$ ($a = 2h$). A plane wave is excited toward the earth, and the antenna scans over the surface of the earth to receive the reflected signals from the interface between the air and the earth. Using the procedure shown in Fig. 5.11, the surface displacements were reconstructed. Figure 5.13 is a graph of the actual displacement and of the displacement calculated for several antenna heights $h$. The displacements are obtained by comparing the phase between the data obtained with and without the displacement in the model. The actual displacement of the surface is a differentiated Gaussian pulse with the maximum amplitude of $1 \mu m$. When the antenna is located at $h = 1$ cm from the surface, the calculated displacement agrees well with the actual displacement. When the antenna is relatively far from the boundary, the displacement obtained from the phase at the observation points is seen to be not a good replica of the actual displacement of the surface.

5.4.2 Simulation of Scanning Focused Antenna

As for the second example, a scanning focused antenna is simulated in the FDTD model to detect the surface displacements. A schematic diagram of the antenna is shown in Fig. 5.14. The observation points are placed at height $h$ above the interface between the air and the earth. A finite number of observation points is arrayed to simulate the focused antenna. The array of observation points is treated as if to focus its recorded signal at a single position on the surface of the earth in the receiving mode. The simulated antenna then scans the surface of the earth to detect the surface displacements in the FDTD grid.
5.4 Detecting Surface Displacements

Figure 5.14: A schematic diagram of the focused antenna simulated in the FDTD model.

Figure 5.15: Actual and computed boundary displacements using the focused antenna when $a = 2h$. 

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5.4 Detecting Surface Displacements

The surface displacements are calculated from the outputs of the focused antenna to demonstrate the effectiveness of detecting surface displacements. Fig. 5.15 is a graph of the actual displacement and of the displacement calculated with the focused antenna when the length of the antenna is two times greater than the antenna height \( h \) \((a = 2h)\). The pulses in the reconstructed displacements are seen to be somewhat wider and smaller than the actual displacement. In general, the reconstructed displacements are quite similar in shape to the actual displacements if compared to Fig. 5.13. It is seen that the reconstructed displacements are not a function of \( h \). This is due to a constant \( a/h \) ratio; the resolution is not sensitive to \( h \) when the \( a/h \) ratio is a constant.

5.4.3 Simulation of Beamforming Array

As for the third example, a beamforming array is simulated in the FDTD model. A schematic diagram of the array is shown in Fig. 5.16. In this model, all of the observation points are arrayed to simulate the beamforming array. In the receiving mode, the beamforming array is treated as if to focus its recorded signal at an arbitrary focal point on the surface of the earth. The focal point is scanned along the surface of the earth so that the array can beamform to detect the surface displacements.

The surface displacements are calculated from the outputs of the beamforming array to demonstrate the effectiveness of detecting surface displacements. Figure 5.17 is a graph of the actual displacement and of the displacement calculated without focusing for several values of the height \( h \) of the array. Here, the displacements are obtained by comparing the phase between the data obtained with and without the displacement in the FDTD grid. The actual displacement of the surface is a differentiated Gaussian pulse with the maximum amplitude of 1 \( \mu m \). When the array is located at \( h = 1 \) cm from the surface, the calculated displacement agrees well with the actual displacement. This demonstrates that when the array is relatively far from the boundary, the displacement obtained from the phase at the array is not a good replica of the actual displacement of the surface.
Figure 5.16: A schematic diagram of the beamforming array in the FDTD model.

In this example, the one-way focusing technique (Eq. (4.2)) was used to obtain a good image of the displacements when the array is relatively far from the surface. The recorded waves at the array with the displacements are summed with the weighting function to focus the array at an arbitrary focal point on the surface. This focal point is scanned over the surface so that the array beamforms over the surface. Figure 5.18 is a graph of the actual displacement and of the displacement calculated with focusing at $h = 20$ cm for several values of $\tau$. The pulses in the reconstructed displacements are seen to be somewhat wider than those in the actual displacement. The pulse is seen to be only slightly wider when $\tau = 30.2\lambda$ and is seen to be much wider when $\tau = 3.8\lambda$. This can be explained using the tradeoff between the beamwidth and the sidelobe illustrated in Fig. 4.8. The beamwidth for $\tau = 30.2\lambda$ is seen to be relatively small and is seen to increase with decreasing values of $\tau$. Thus, the increased width of the pulse is seen to be due to the increased beamwidth of the array. A ripple is seen on the reconstructed displacements. The amplitude of the ripple is seen to increase with increasing $\tau$. This is because the sidelobe levels increase with increasing $\tau$ as shown in Fig. 4.8.
Figure 5.17: Actual and computed boundary displacements without beamforming.

Figure 5.18: Actual and computed boundary displacements with beamforming when $h = 20$ cm.
5.4 Detecting Surface Displacements

5.4.4 Sensitivity

Very small surface displacements were successfully reconstructed in the FDTD model in the previous section. For the model, the source code was written in the FORTRAN 90 programming language, and double precision was used. This section discusses the sensitivity of the FDTD model to the smallest possible amplitude of the displacements detectable in the model. The maximum displacement obtained from the FDTD model is graphed as a function of the actual displacement of the surface in Fig. 5.19. The graph is plotted in log scale. Here, the maximum amplitude of a differentiated Gaussian displacement was varied in the range of $10^{-4}$ m $\sim 10^{-18}$ m, and the displacement was reconstructed at height $h = 1$ cm for the observation points. For displacements greater than approximately $10^{-18}$ m, the FDTD model is accurate; however, for displacements less than approximately $10^{-18}$ m, the FDTD model is not accurate.
CHAPTER 6

Comparisons

In this section, some comparisons have been made for the antenna configurations investigated in this research. The configurations include (1) a focused antenna, (2) a synthetic beamforming array, (3) a physical array of N antennas using simple antennas. Comparisons are made in terms of the advantages and the disadvantages of each configuration and summarized in Table 6.1. Important factors such as spatial resolution and scanning speed are explained below for each configuration. Note that for all of the configurations, a sufficient spatial resolution can be obtained at a standoff distance.

**Focused Antenna:** A sufficient spatial resolution can be obtained at a standoff distance for the radar. Moreover, no post-processing is needed for the measured data. However, when a single focused antenna is used for the radar, a large amount of scanning time is required. The scanning time can be reduced by using a physical array of N focused antennas, which allows reduction of the scanning time by a factor of N. Ideally, one would like for the focused spot on the ground to be adjacent; then, the array would only have to be scanned in one direction. However, the focused spots for this array are not adjacent due to the physical size of the antennas (see Fig. 6.1).

**Synthetic Beamforming Array:** This configuration provides a simple structure for the system because it uses a single simple antenna. However, this configuration requires a large amount of scanning time and post-processing to beamform the measured data.
Figure 6.1: Spaces between the focused spots of the neighboring focused antennas.

**Physical Array of N Antennas (One-Way Focus):** Although it uses simple antennas, the beauty of this configuration arises from the high speed with which the radar scans. This is possible because a focusing algorithm can be used in both the software and hardware. The disadvantages of this configuration are expense and complexity. In addition, neighboring antennas will create cross talk, and the plane wave excitation may be difficult.

**Physical Array of N Antennas (Two-Way Focus):** This configuration will allow smaller spatial resolution than the others when many antennas are used to populate the array; the array focuses in the two-way mode with a large aperture dimension for the array. However, a hardware beamformer must be used to reduce the scanning time, which may result in a complicated system.
Table 6.1: Advantages and disadvantages of five antenna configurations investigated as means to detect surface displacement.

<table>
<thead>
<tr>
<th>Antenna configuration</th>
<th>Advantage</th>
<th>Disadvantage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Focused antenna</td>
<td>Spatial resolution</td>
<td>Slow scanning speed</td>
</tr>
<tr>
<td></td>
<td>No post-processing</td>
<td></td>
</tr>
<tr>
<td>Physical array of N focused antennas</td>
<td>Spatial resolution</td>
<td>Space between spots</td>
</tr>
<tr>
<td></td>
<td>No post-processing</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fast scanning speed</td>
<td></td>
</tr>
<tr>
<td>Synthetic beamforming array</td>
<td>Spatial resolution</td>
<td>Slow scanning speed</td>
</tr>
<tr>
<td></td>
<td>Simple structure</td>
<td>Post-processing</td>
</tr>
<tr>
<td>Physical array of N antennas: one-way</td>
<td>Spatial resolution</td>
<td>Interaction between antennas</td>
</tr>
<tr>
<td>focus</td>
<td>Fast scanning speed</td>
<td>Plane wave excitation</td>
</tr>
<tr>
<td></td>
<td>Beamforming in hardware</td>
<td></td>
</tr>
<tr>
<td></td>
<td>or in software</td>
<td></td>
</tr>
<tr>
<td>Physical array of N antennas:</td>
<td>Spatial resolution</td>
<td>Interaction between antennas</td>
</tr>
<tr>
<td>two-way focus</td>
<td>Fast scanning speed</td>
<td>Potentially complex</td>
</tr>
<tr>
<td></td>
<td>Beamforming in hardware</td>
<td>Beams forming in hardware</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
CHAPTER 7

Summary and Conclusions

The objective of this research was to investigate methods for improving the performance of a radar that is intended to be used for a mine detection system that uses elastic and electromagnetic waves simultaneously and to solve some of the current limitations of the radar system: small standoff distance and slow scanning speed.

First, a basic theory of the focused aperture was studied. Here, properties of near-field focusing were investigated. The radiation pattern of the aperture was computed to examine the behaviors of the spot size and the sidelobe level of the patterns. These behaviors were studied as a function of the focal distance and the dimension of the aperture. It was observed that the smallest spot size does not occur at the focal plane when the aperture is placed in the near-field region; it occurs at an intermediate location between the aperture and the focal plane.

For a practical focused antenna, a lens-focused conical corrugated horn antenna was developed and built for use in the measurement of surface displacements. This antenna can provide a sufficient spatial resolution for the radar. The antenna consists of a conical corrugated horn and a dielectric lens with two foci. This antenna was analyzed theoretically. The analysis included the radiation pattern (one-way pattern) of the antenna and the behavior of the spot size of the pattern. Also, a concept of the two-way pattern was introduced. The two-way pattern was studied because the antenna functions as both a transmitter and a receiver for the radar. Numerical results for the theoretical model of the antenna showed the properties of the radiation pattern of this antenna are, as expected, similar to those of the focused aperture, and
the two-way pattern is seen to be the square of the one-way pattern.

Prototypes of the lens-focused corrugated horn were designed and built for use with the radar; four dielectric lenses and four corrugated horns have been manufactured. The performance of the prototypes was also verified with a series of experiments. A modulating scatterer was used to measure the two-way pattern of the antenna. The measured two-way pattern was in good agreement with the theoretically computed pattern. Finally, this antenna was implemented with the land mine detection system and tested in laboratory experiments. Measured results showed that a sufficient spatial resolution can be obtained so that surface displacements can be measured, thus detecting a mine when the antenna is placed farther from the surface.

Next, a synthetic beamforming array was studied as an alternative technique to obtain the required spatial resolution with the required standoff. A simple antenna was used for the radar, and the antenna was scanned to construct a synthetic array. A theoretical model of the synthetic beamforming array was developed. It simulated the signals received by the antenna due to the displacement of the surface, and the beamforming technique was then implemented to reconstruct the surface displacement. Moreover, the received pattern was synthesized for the array to investigate the beamwidth and the sidelobe levels of the pattern. It was found that there is a tradeoff between the beamwidth and the sidelobe levels. In addition, several experiments were performed to investigate the feasibility of the array; the synthetic beamforming array was used to detect both static and dynamic surface displacements. In both measurements, the spatial resolution of the radar was improved when the beamforming technique was used.

A study was made of a physical array of $N$ antennas for the purpose of improving the scanning speed as well as creating a greater standoff distance for the radar. Using a simple antenna such as an isotropic or an aperture antenna, two physical array configurations were studied: one-way focus and two-way focus. Theoretical models were developed for both one-way and two-way focusing arrays. The theoretical models were based on the integral equation method. For both one-way and two-way focusing
arrays, appropriate weighting functions to focus the arrays at an arbitrary focal point were chosen, and the received patterns of the arrays were computed to examine the spatial resolution (beamwidth) of the array.

Then, a two-dimensional finite-difference time-domain (FDTD) model was developed to examine the feasibility of detecting surface displacements and to help to understand part of the above sensor techniques. In this model, a total field/scattered field formulation was used to inject an electromagnetic plane wave, a uniaxial perfectly matched layer was used to truncate the boundaries, and the earth was modeled as a lossy half space. The surface of the earth was modeled to be displaced in the shape of a differentiated Gaussian pulse. These surface displacements were reconstructed by simulating two techniques: (1) a scanning focused antenna and (2) a beamforming array. Numerical results showed that very small surface displacements could be reconstructed successfully using those methods in the FDTD simulation.

Finally, some comparisons were made for the different antenna configurations studied in this work. Comparisons were made in terms of the advantages and the disadvantages of each technique. Important factors such as spatial resolution and scanning speed were explained for each technique.

All of the antenna configurations are promising. A sufficient spatial resolution can be obtained at a standoff distance for the radar, and the scanning speed can be improved by using a physical array of N focused antennas or of N simple antennas with an appropriate beamforming technique.

Future research associated with this topic could be to actually build a physical array of N antennas and to test the array both in laboratory experiments and in a real mine field. However, one must investigate priori issues associated with the array. The priori issues would be how to deal with the mutual coupling between antennas, antenna selection, and how to feed the elements.
APPENDIX A

Derivations of Field Equations Associated with Current Sources

This appendix derives the equations for $E$- and $H$-fields produced by current densities on the surface of an arbitrary volume. The $E$-field is computed by the direct integration of the field equations. The derivation is started with Maxwell's equations. At the end, the derivation ends up with Eqs.(2.10) and (2.11). Furthermore, more facilitated forms (Eqs.(2.12) and (2.13)) of Eqs.(2.10) and (2.11) were derived for a particular coordinate system instead of using generalized formulations.

Consider an arbitrary volume $v$ enclosed by a surface $s$ as shown in Fig. A.1. A surface current density $\vec{J}$ is assumed to exist on the infinitesimal surface element $ds$. A outward unit normal vector is also defined on the surface element $ds$. Figure A.2 shows the coordinate system for this problem. Here, four vectors and vector operators are defined as the following:

$$\vec{R} = x\hat{x} + y\hat{y} + z\hat{z},$$

Observer vector,

$$\vec{R}' = x'\hat{x} + y'\hat{y} + z'\hat{z},$$

Source vector,

$$\vec{r} = \vec{R} - \vec{R}' = (x - x')\hat{x} + (y - y')\hat{y} + (z - z')\hat{z},$$

$$r = |\vec{r}| = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2},$$

$$\hat{r} = \frac{\vec{r}}{r},$$

$$\nabla = \hat{x} \frac{\partial}{\partial x} + \hat{y} \frac{\partial}{\partial y} + \hat{z} \frac{\partial}{\partial z}, \quad (A.1)$$

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Figure A.1: Current densities on an arbitrary volume.

\[ \nabla' = \hat{x} \frac{\partial}{\partial x'} + \hat{y} \frac{\partial}{\partial y'} + \hat{z} \frac{\partial}{\partial z'}. \]  

(A.2)

In general, \( \nabla f(r) = -\nabla' f(r) \).

Starting with

\[ \vec{E}_e = -\nabla \Phi - j\omega \vec{A}, \]  

(A.3)

\[ \vec{H}_e = \nabla \times \vec{A}/\mu_0, \]  

(A.4)

where

\[ \Phi = \frac{1}{4\pi \epsilon_0} \int_{v'} \rho \Psi dv', \]

\[ \vec{A} = \frac{\mu_0}{4\pi} \int_{v'} \vec{J} \Psi dv', \]

\[ \Psi = e^{-jk_0r}/r. \]

Therefore,

\[ \vec{H}_e = \nabla \times \left( \frac{1}{\mu_0} \cdot \frac{\mu_0}{4\pi} \int_{v'} \vec{J} \Psi dv' \right) \]

\[ = \frac{1}{4\pi} \int_{v'} \nabla \times (\vec{J} \Psi) dv'. \]  

(A.5)
Derivations of Field Equations Associated with Current Sources

Figure A.2: A coordinate system of the current source.

From the vector identity,
\[ \nabla \times (\vec{J} \Psi) = \Psi \nabla \times \vec{J} + \vec{\nabla} \times \vec{J} \]
\[ = \vec{\nabla} \times \vec{J}. \]

Thus, Eq. (A.5) becomes
\[ \vec{H}_e = \frac{1}{4\pi} \int_{v'} -\vec{J} \times \nabla \Psi dv'. \]  \hspace{1cm} (A.6)

It has been found that \( \nabla \Psi = -\nabla' \Psi \), therefore,
\[ \vec{H}_e = \frac{1}{4\pi} \int_{v'} \vec{J} \times \nabla' \Psi dv'. \]  \hspace{1cm} (A.7)

For \( \vec{E}_e \), using Lorentz condition (or Lorentz gauge) for potentials, \( \Phi \) can be given by
\[ \Phi = \frac{\nabla \cdot \vec{A}}{-j\omega \mu_o \epsilon_o}. \]  \hspace{1cm} (A.8)

Then,
\[ \vec{E}_e = \frac{1}{j\omega \mu_o \epsilon_o} \nabla (\nabla \cdot \vec{A}) - j\omega \vec{A} \]
\[ = \frac{1}{j\omega \mu_o \epsilon_o} \nabla \left\{ \frac{\mu_o}{4\pi} \int_{v'} \nabla \cdot (\vec{J} \Psi) dv' \right\} - j\omega \frac{\mu_o}{4\pi} \int_{v'} \vec{J} \Psi dv'. \]  \hspace{1cm} (A.9)
Table A.1: Pairs of duality principle.

<table>
<thead>
<tr>
<th>( E_e \leftrightarrow H_m )</th>
<th>( J \leftrightarrow M )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_e \leftrightarrow -E_m^\prime )</td>
<td>( k_o \leftrightarrow k_o )</td>
</tr>
<tr>
<td>( \epsilon_o \leftrightarrow \mu_o )</td>
<td>( \mu_o \leftrightarrow \epsilon_o )</td>
</tr>
</tbody>
</table>

Using vector identities,

\[
\nabla \cdot (\vec{J}\Psi) = \vec{J} \cdot \nabla \Psi + \nabla \cdot \vec{J} = \vec{J} \cdot \nabla \Psi,
\]

\[
\nabla (\vec{J} \cdot \nabla \Psi) = (\vec{J} \cdot \nabla)\nabla \Psi + \nabla \times (\nabla \times \vec{J}) + \nabla \times (\nabla \times \vec{J}) = (\vec{J} \cdot \nabla)\nabla \Psi.
\]

Eq. (A.9) becomes

\[
\vec{E}_e = \frac{1}{j\omega \epsilon_o} \cdot \frac{1}{4\pi} \int \left( \vec{J} \cdot \nabla \right) \nabla \Psi \, dv' - \frac{j\omega \mu_o}{4\pi} \int \vec{J} \Psi \, dv'.
\] (A.10)

Also, using \((\vec{J} \cdot \nabla)\nabla \Psi = (\vec{J} \cdot \nabla')\nabla' \Psi\), finally,

\[
\vec{E}_e = \frac{-j\omega \mu_o}{4\pi} \int \left\{ \vec{J} \Psi + \frac{1}{k_o} (\vec{J} \cdot \nabla') \nabla' \Psi \right\} dv'.
\] (A.11)

Equations (A.11) and (A.7) can be directly used to obtain the \( E \)- and \( H \)-fields due to an electric current density. However, when a magnetic current density \((\vec{M})\) simultaneously exists on the volume, the radiated field equations must be modified to include the radiation due to the magnetic source. The fields due to the magnetic source can be easily obtained using the duality principle [46]. This principle is tabulated in Table A.1. Here, \( \vec{E}_m \) and \( \vec{H}_m \) denote the \( E \)- and \( H \)-fields due to the pure magnetic source. Then, a transformed set of electromagnetic field equations can be formulated using Table A.1. Then, a complete set of the field equations due to the
Figure A.3: Coordinate systems for the source and observation positions.

The electric and magnetic sources is given by

\[
\vec{E}(\vec{r}) = \frac{-j \omega \mu_0}{4\pi} \int \left[ \vec{J} \Psi + \frac{1}{k_0^2} (\vec{J} \cdot \nabla') \nabla' \Psi \right] dv' - \frac{1}{4\pi} \int (\vec{M} \times \nabla' \Psi) dv',
\]

\[
\vec{H}(\vec{r}) = \frac{-j \omega \epsilon_0}{4\pi} \int \left[ \vec{M} \Psi + \frac{1}{k_0^2} (\vec{M} \cdot \nabla') \nabla' \Psi \right] dv' + \frac{1}{4\pi} \int (\vec{J} \times \nabla' \Psi) dv'.
\]

In order to integrate these equations, they must be facilitated by vector calculus.

Consider following two coordinate systems shown in Fig. A.3. Here, \( \hat{\rho} = -\hat{r} \) and \( r = \rho = |r| = |\rho| \). Thus, \( \Psi = \frac{e^{-jk_o \rho}}{\rho} \), which is only a function of \( \rho \). Therefore,

\[
\nabla' \Psi = \hat{\rho} \frac{\partial \Psi}{\partial \rho} + \hat{\theta} \frac{1}{\rho} \frac{\partial \Psi}{\partial \theta} + \hat{\phi} \frac{1}{\rho \sin \theta} \frac{\partial \Psi}{\partial \phi},
\]

and

\[
\nabla' \Psi = \frac{e^{-jk_o \rho}}{\rho} \left( -jk_o - \frac{1}{\rho} \right) \hat{\rho}.
\]

Using \( \rho = r \) and \( \hat{\rho} = -\hat{r} \),

\[
\nabla' \Psi = \left( jk_o + \frac{1}{r} \right) \frac{e^{-jk_o r}}{r} \hat{r}.
\]

Next, \( \vec{J} \) can be divided into each coordinate component; \( \vec{J} = J_\rho \hat{\rho} + J_\theta \hat{\theta} + J_\phi \hat{\phi} \). Then,

\[
\vec{J} \cdot \nabla' = J_\rho \frac{\partial}{\partial \rho} + J_\theta \frac{1}{\rho} \frac{\partial}{\partial \theta} + J_\phi \frac{1}{\rho \sin \theta} \frac{\partial}{\partial \phi}.
\]
Derivations of Field Equations Associated with Current Sources

From Eq. (A.14),

\[ (\hat{\mathbf{J}} \cdot \nabla') \nabla' \Psi = \left( J_\rho \partial_{\rho} \left[ \frac{e^{-jk_o \rho}}{\rho} \right] - \frac{1}{\rho} \right) \rho \partial_{\rho} \left[ \frac{e^{-jk_o \rho}}{\rho} \right] + J_\theta \frac{1}{\rho} \partial_{\theta} \left[ \frac{e^{-jk_o \rho}}{\rho} \right] \]

Term A

\[ + J_\phi \frac{1}{\rho \sin \theta} \partial_{\phi} \left[ \frac{e^{-jk_o \rho}}{\rho} \right] \]  

Term B

\[ + \frac{\partial}{\partial \rho} \left[ \frac{e^{-jk_o \rho}}{\rho} \right] \frac{\partial}{\partial \rho} \rho \]  

Term C

\[ + \frac{\partial}{\partial \rho} \left[ \frac{e^{-jk_o \rho}}{\rho} \right] \frac{\partial}{\partial \rho} \rho \]  

The first term A is then

\[ A = J_\rho \left[ \frac{2}{\rho^3} j k_o e^{-jk_o \rho} - k_o e^{-jk_o \rho} \rho \right] \]

and for the second term B,

\[ B = J_\theta \left[ \frac{1}{\rho} \partial_{\rho} \left[ \frac{e^{-jk_o \rho}}{\rho} \right] \right] \theta. \]

For the third term C, using \( \frac{\partial}{\partial \rho} = \phi \sin \theta, \)

\[ C = J_\phi \left[ \frac{1}{\rho} \left[ \frac{e^{-jk_o \rho}}{\rho} \right] \right] \phi. \]

Also, note that \( J_\rho \hat{\rho} = (\hat{\mathbf{J}} \cdot \hat{\rho}) \hat{\rho}, \) \( \hat{\rho} = -\hat{\mathbf{r}}, \) and \( |r| = |ho|. \) Thus, Eq. (A.16) becomes

\[ (\hat{\mathbf{J}} \cdot \nabla') \nabla' \Psi = \left[ - \frac{k_o^2}{2} \left( \hat{\mathbf{J}} \cdot \hat{\mathbf{r}} \right) \hat{r} + \frac{3}{r} (j k_o + \frac{1}{r}) \left( \hat{\mathbf{J}} \cdot \hat{\mathbf{r}} \right) \hat{r} - \frac{\hat{J}}{r} (j k_o + \frac{1}{r}) \right] \Psi. \]  

(A.17)

Finally, plugging Eqs. (A.15) and (A.17) into Eq. (A.12), Eq. (A.12) can be written as

\[ \hat{E}(\hat{R}) = \frac{-j \omega \mu_0}{4\pi} \int_{\mathbb{V}} \left[ \hat{J} + \frac{1}{k^2} \left[ - \frac{k_o^2}{2} \left( \hat{\mathbf{J}} \cdot \hat{\mathbf{r}} \right) \hat{r} + \frac{3}{r} (j k_o + \frac{1}{r}) \left( \hat{\mathbf{J}} \cdot \hat{\mathbf{r}} \right) \hat{r} - \frac{\hat{J}}{r} (j k_o + \frac{1}{r}) \right] \right] \]

\[ \times \frac{e^{-jk_o r}}{r} dv' - \frac{1}{4\pi} \int_{\mathbb{V}} \left[ \hat{M} \times \hat{r} (j k_o + \frac{1}{r}) e^{-jk_o r} \right] \frac{1}{r} dv', \]  

(A.18)
Also, using the duality principle again,

\[
\tilde{H}(\tilde{r}) = \frac{-j\omega\epsilon_0}{4\pi} \int_{\mathcal{V}} \{ \tilde{M} + \frac{1}{k_0^2} [-k_0^2 (\tilde{M} \cdot \hat{r}) \hat{r} + \frac{3}{r} (j k_0 + \frac{1}{r})(\tilde{M} \cdot \hat{r}) \hat{r} - \frac{\tilde{M}}{r} (j k_0 + \frac{1}{r})] \}
\]

\[
\times \frac{e^{-j k_o r}}{r} dv' + \frac{1}{4\pi} \int_{\mathcal{V}} [\tilde{J} \times \hat{r} (j k_0 + \frac{1}{r})] \frac{e^{-j k_o r}}{r} dv'.
\]  \quad \text{(A.19)}

No approximations such as far-field approximation have been used in the above equations. These equations exactly represent the relationships between existing sources and the fields radiated by the sources. Therefore, these equations are valid in all field regions (near-field or far-field). However, if the observation point is located at the source position, these equations fail. At that location, there exists a singular point that causes the integrand to be infinite. Therefore, the observation point should not be located at the source position.
APPENDIX B

2-D FDTD Update Equations for an Anisotropic PML

This appendix includes the 2-D FDTD update equations for an anisotropic perfectly matched layer (PML) for the TM set.

A schematic diagram of the PML structure in the two-dimensions is shown in Fig. B.1. The region of interest is a lossy and non-dispersive medium with \( \epsilon_r \), \( \mu_o \), and \( \sigma \). The PML surrounds the region of interest on the four sides (top, bottom, left, and right), and there exist four corners where there is more than one normal interface boundary. The PML update equations for each field component are slightly different from each other depending on the computation region. The update equations, for example, are much more complicated at the four corner regions. Detailed derivations of the PML update equations can be found in [37]. Here, samples of the update equations are given for all field components for the TM set.

For \( H_z \) for the top, bottom, left, and right faces of the PML,

\[
H_z^{n+1/2}(i,j) = \left[ \frac{2\epsilon_o - \Delta t \sigma}{2\epsilon_o + \Delta t \sigma} \right] H_z^{n-1/2}(i,j) - \frac{1}{\Delta x \mu_o \mu_r} \left[ \frac{2\epsilon_o \Delta t}{2\epsilon_o + \sigma \Delta t} \right] \\
\times \left[ E_y^n(i+1/2,j) - E_y^n(i-1/2,j) \right] + \frac{1}{\Delta y \mu_o \mu_r} \\
\times \left[ \frac{2\epsilon_o \Delta t}{2\epsilon_o + \sigma \Delta t} \right] \left[ E_x^n(i,j+1/2) - E_x^n(i,j-1/2) \right]. \tag{B.1}
\]

For \( E_x \) for the left and right faces of the PML,

\[
E_x^{n+1}(i,j+1/2) = E_x^n(i,j+1/2) + \left[ \frac{2\epsilon_o + \sigma_x \Delta t}{2\epsilon_o} \right] D_x^{n+1}(i,j+1/2)
\]

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2-D FDTD Update Equations for an Anisotropic PML

\[ - \left[ \frac{2\epsilon_o - \sigma_x \Delta t}{2\epsilon_o} \right] D_x^n(i, j + 1/2), \]

where

\[ D_x^{n+1}(i, j + 1/2) = \left[ \frac{2\epsilon_o \epsilon_r - \sigma \Delta t}{2\epsilon_o \epsilon_r + \sigma \Delta t} \right] D_x^n(i, j + 1/2) + \frac{1}{\Delta y} \left[ \frac{2\Delta t}{2\epsilon_o \epsilon_r + \sigma \Delta t} \right] \times \left[ H_x^{n+1/2}(i, j + 1) - H_x^{n+1/2}(i, j) \right]. \]  

For \( E_y \) for the left and right faces of the PML,

\[ E_y^{n+1}(i + 1/2, j) = \left[ \frac{2\epsilon_o - \sigma_x \Delta t}{2\epsilon_o + \sigma_x \Delta t} \right] E_y^n(i + 1/2, j) + \left[ \frac{2\epsilon_o}{2\epsilon_o + \sigma_x \Delta t} \right] \times \left[ D_y^{n+1}(i + 1/2, j) - D_y^n(i + 1/2, j) \right], \]

where

\[ D_y^{n+1}(i + 1/2, j) = \left[ \frac{2\epsilon_o \epsilon_r - \sigma \Delta t}{2\epsilon_o \epsilon_r + \sigma \Delta t} \right] D_y^n(i + 1/2, j) - \frac{1}{\Delta x} \left[ \frac{2\Delta t}{2\epsilon_o \epsilon_r + \sigma \Delta t} \right] \times \left[ H_x^{n+1/2}(i + 1, j) - H_x^{n+1/2}(i, j) \right]. \]

For \( E_x \) for the top and bottom of the PML,

\[ E_x^{n+1}(i, j + 1/2) = \left[ \frac{2\epsilon_o - \sigma_y \Delta t}{2\epsilon_o + \sigma_y \Delta t} \right] E_x^n(i, j + 1/2) + \left[ \frac{2\epsilon_o}{2\epsilon_o + \sigma_y \Delta t} \right] \]
\[ \times \left[ D_{x}^{n+1}(i, j + 1/2) - D_{x}^{n}(i, j + 1/2) \right], \quad (B.6) \]

where
\[
D_{x}^{n+1}(i, j + 1/2) = \left[ \frac{2\varepsilon_{o}\varepsilon_{r} - \sigma_{z}\Delta t}{2\varepsilon_{o}\varepsilon_{r} + \sigma_{z}\Delta t} \right] D_{z}^{n}(i, j + 1/2) + \frac{1}{\Delta y} \left[ \frac{2\Delta t}{2\varepsilon_{o}\varepsilon_{r} + \sigma_{z}\Delta t} \right] \times \left[ H_{z}^{n+1/2}(i, j + 1) - H_{z}^{n+1/2}(i, j) \right]. \quad (B.7) \]

For \( E_{y} \) for the left and right faces of the PML,
\[
E_{y}^{n+1}(i + 1/2, j) = E_{y}^{n}(i + 1/2, j) + \frac{[2\varepsilon_{o} + \sigma_{y}\Delta t]}{2\varepsilon_{o}} D_{y}^{n+1}(i + 1/2, j) \]
\[
- \frac{[2\varepsilon_{o} - \sigma_{y}\Delta t]}{2\varepsilon_{o}} D_{y}^{n}(i + 1/2, j), \quad (B.8) \]

where
\[
D_{y}^{n+1}(i + 1/2, j) = \left[ \frac{2\varepsilon_{o}\varepsilon_{r} - \sigma_{z}\Delta t}{2\varepsilon_{o}\varepsilon_{r} + \sigma_{z}\Delta t} \right] D_{y}^{n}(i + 1/2, j) - \frac{1}{\Delta x} \left[ \frac{2\Delta t}{2\varepsilon_{o}\varepsilon_{r} + \sigma_{z}\Delta t} \right] \times \left[ H_{z}^{n+1/2}(i + 1, j) - H_{z}^{n+1/2}(i, j) \right]. \quad (B.9) \]

For \( H_{z} \) for the four corners of the PML,
\[
H_{z}^{n+1/2}(i, j) = \frac{[2\varepsilon_{o} - \Delta t\sigma_{x}]}{2\varepsilon_{o} + \Delta t\sigma_{x}} H_{z}^{n-1/2}(i, j) + \frac{1}{\mu_{o}\mu_{r}} \left[ \frac{2\varepsilon_{o}}{2\varepsilon_{o} + \sigma_{x}\Delta t} \right] \times \left[ B_{z}^{n+1/2}(i, j) - B_{z}^{n-1/2}(i, j) \right], \quad (B.10) \]

where
\[
B_{z}^{n+1/2}(i, j) = \left[ \frac{2\varepsilon_{o} - \Delta t\sigma_{y}}{2\varepsilon_{o} + \Delta t\sigma_{y}} \right] B_{z}^{n-1/2}(i, j) - \left[ \frac{2\Delta t\varepsilon_{o}}{2\varepsilon_{o} + \Delta t\sigma_{y}} \right] \times \left[ \frac{E_{y}^{n}(i + 1/2, j) - E_{y}^{n}(i - 1/2, j)}{\Delta x} \right] \]
\[
- \left[ \frac{E_{y}^{n}(i, j + 1/2) - E_{y}^{n}(i, j - 1/2)}{\Delta y} \right]. \quad (B.11) \]
2-D FDTD Update Equations for an Anisotropic PML

For $E_x$ for the four corners of the PML,

$$E_{x}^{n+1}(i, j + 1/2) = [\frac{2\varepsilon_0 \varepsilon_r - \Delta t \sigma_x}{2\varepsilon_0 \varepsilon_r + \Delta t \sigma_x}] E_{x}^{n}(i, j + 1/2) + \left[ \frac{2\Delta t}{2\varepsilon_0 \varepsilon_r + \Delta t \sigma_x} \right]$$

$$\times \left[ D_{x}^{n+1}(i, j + 1/2) \left[ \frac{2\varepsilon_0 + \sigma_x \Delta t}{2\varepsilon_0 \Delta t} \right] \right]$$

$$- D_{x}^{n}(i, j + 1/2) \left[ \frac{2\varepsilon_0 - \sigma_x \Delta t}{2\varepsilon_0 \Delta t} \right], \quad (B.12)$$

where

$$D_{x}^{n+1}(i, j + 1/2) = [\frac{2\varepsilon_0 - \Delta t \sigma_y}{2\varepsilon_0 + \Delta t \sigma_y}] D_{x}^{n}(i, j + 1/2) + \frac{1}{\Delta y} \left[ \frac{2\Delta t \varepsilon_0}{2\varepsilon_0 + \Delta t \sigma_y} \right]$$

$$\times \left[ H_{z}^{n+1/2}(i, j + 1) - H_{z}^{n+1/2}(i, j) \right], \quad (B.13)$$

For $E_y$ for the four corners of the PML,

$$E_{y}^{n+1}(i + 1/2, j) = [\frac{2\varepsilon_0 \varepsilon_r - \Delta t \sigma_y}{2\varepsilon_0 \varepsilon_r + \Delta t \sigma_y}] E_{y}^{n}(i + 1/2, j) + \left[ \frac{2\Delta t}{2\varepsilon_0 \varepsilon_r + \Delta t \sigma_y} \right]$$

$$\times \left[ D_{y}^{n+1}(i + 1/2, j) \left[ \frac{2\varepsilon_0 + \sigma_y \Delta t}{2\varepsilon_0 \Delta t} \right] \right]$$

$$- D_{y}^{n}(i + 1/2, j) \left[ \frac{2\varepsilon_0 - \sigma_y \Delta t}{2\varepsilon_0 \Delta t} \right], \quad (B.14)$$

where

$$D_{y}^{n+1}(i + 1/2, j) = [\frac{2\varepsilon_0 - \Delta t \sigma_x}{2\varepsilon_0 + \Delta t \sigma_x}] D_{y}^{n}(i + 1/2, j) - \frac{1}{\Delta x} \left[ \frac{2\varepsilon_0 \Delta t}{2\varepsilon_0 + \Delta t \sigma_x} \right]$$

$$\times \left[ H_{z}^{n+1/2}(i + 1, j) - H_{z}^{n+1/2}(i, j) \right], \quad (B.15)$$
APPENDIX C

Surface Matched Lens by Simulating a Quarter-Wavelength Matching Layer

This appendix includes details on designing a surface matched lens using horizontal corrugations. These corrugations are perpendicular with respect to the polarization of the exciting $E$-field.

Electromagnetic waves incident on any ordinary dielectric lens with a dielectric constant other than unity will be reflected from the surface of the lens and will undergo multiple reflections within the lens. This results in a high input standing-wave ratio, the loss of the input power, and various anomalies in the radiated field from the lens. The degree of the reflection is a function of the incident angle, the dielectric constant, and the polarization of the incident wave. This surface reflection can be eliminated by coating the surface with an additional dielectric layer with a permittivity that is the mean of the permittivities on the two sides of the interface (See Fig. C.1). The layer must be approximately a quarter-wavelength in depth so that the reflected field can be cancelled out with the incident field. Several techniques for simulating a quarter-wavelength layer have been investigated by various groups and can be found in [27], [47], and [48]. These techniques include vertical and horizontal corrugations, the array of the dielectric cylinders, and the array of holes in the dielectric. In this work, the use of horizontal corrugations on the lens surface has been selected, and the design details shown below are from [48].

Figure C.2 illustrates horizontal corrugations on the lens surfaces. The corrugations are oriented perpendicular to the incident $E$-field. The parameters to be
Figure C.1: Anti-reflection matching layer.

determined are the depth, \( d \), the thickness, \( t \), and the spacing, \( D \), of the corrugation. For an arbitrary curved surface of the lens, these parameters depend on the angle of incidence, \( \theta_i \), of the incident field and its polarization. The required equivalent permittivity for the matching layer is given by

\[
\varepsilon_x = \sin^2 \theta_i + \cos \theta_i \sqrt{\varepsilon_r - \sin^2 \theta_i} \tag{C.1}
\]

for a perpendicular polarization to the plane of incident, and

\[
\varepsilon_x = \frac{1 + \sqrt{1 - 4\alpha \sin^2 \theta_i}}{2\alpha}, \tag{C.2}
\]

where \( \alpha = \cos \theta_i \sqrt{\varepsilon_r - \sin^2 \theta_i}/\varepsilon_r \) for a parallel polarization to the plane of incident. The depth, \( d \), of the corrugation must be

\[
d = \frac{\lambda_o}{4\sqrt{\varepsilon_x - \sin^2 \theta_i}}, \tag{C.3}
\]

where \( \lambda_o \) is the free space wavelength.

The equivalent permittivity of the matching layer can be obtained from an adequate choice of the ratio of \( t \) and \( D \). The first-order approximations for the ratio of \( t \) and \( D \) have been found to be [48]

\[
\varepsilon_x = 1 + (\varepsilon_r - 1) \frac{t}{D} \tag{C.4}
\]
for perpendicular polarization, and

\[
\varepsilon_x = \frac{1}{1 - \frac{1}{\varepsilon_r} \frac{r}{D}}
\]  

(C.5)

for parallel polarization. From these relations, an equivalent permittivity can be obtained between 1 and \( \varepsilon_r \).

\( D \) determines the number of corrugations. Therefore, for the minimum number of the corrugations, large spacings, \( D \), are required. High-order grating modes, however, must be avoided in the lens. Thus, \( D \) must be limited by the relation \( D < \lambda_o/(\sqrt{\varepsilon_r} + \sin \theta) \) [27].

As an example, a surface matched lens was designed with horizontal corrugations. The parameters for the lens are shown in Table C.1. The \( E \)-field was assumed to be excited at the location of \( F_1 \). \( D \) was chosen to be a constant; \( D = \lambda_o/(2\sqrt{\varepsilon_r}) \). Using Eqs. (C.2) and (C.5) and \( D, t \) was determined. Figure C.3 is a 3-D graph of the lens designed. The corrugations must be normal to the surface of the lens.

These lenses may be difficult to manufacture using traditional technique. However, rapid prototyping machines such as Stereolithography (SLA), Selective Laser
Table C.1: Predetermined parameters for the surface matched lens.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter of the lens</td>
<td>20 cm</td>
</tr>
<tr>
<td>Two foci, $F_1$ and $F_2$</td>
<td>20 cm and 30 cm</td>
</tr>
<tr>
<td>Dielectric constant of the lens, $\varepsilon_r$</td>
<td>2.94</td>
</tr>
<tr>
<td>Operating frequency, $f$</td>
<td>8 GHz</td>
</tr>
<tr>
<td>Polarization of the excited wave</td>
<td>Linear</td>
</tr>
</tbody>
</table>

Sintering (SLS), or Fused Deposition Modeling (FDM) can be used to avoid manufacturing difficulties, but each machine can only work with certain materials.
Figure C.3: Surface matched lens with horizontal corrugations.
Bibliography


VITA

Seung-Ho Lee was born in Seoul, South Korea, on June 27, 1970. He received the B.E. degree in electrical engineering from the Hanyang University, South Korea in 1993, and the M.S. degree in electrical engineering from the University of Southern California, Los Angeles, CA in 1996. Since 1996, he has been working toward the Ph.D. degree at the Georgia Institute of Technology. Since 1997, he has been a Graduate Research Assistant in the School of Electrical and Computer Engineering at the Georgia Institute of Technology. His research interests include antenna design, computational electromagnetics, and ground penetrating radar systems.